Proving Information Flow Security for Concurrent Programs

Marco Eilers
Thibault Dardinier
Peter Müller

ETH zürich

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(Automatic) Program Verification

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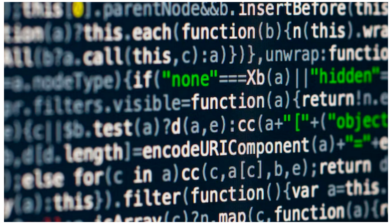

Source code
(e.g., sort algorithm)

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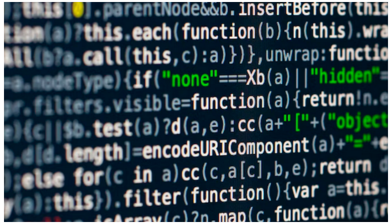

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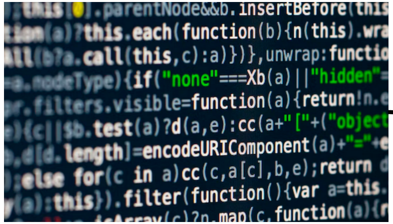

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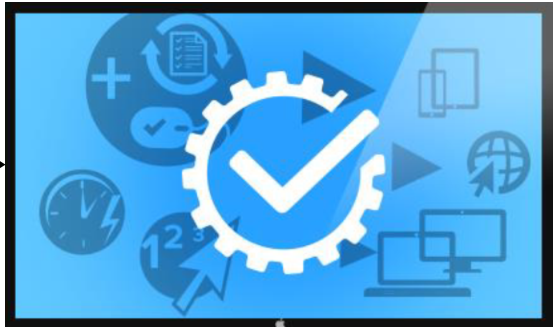

Program verifier

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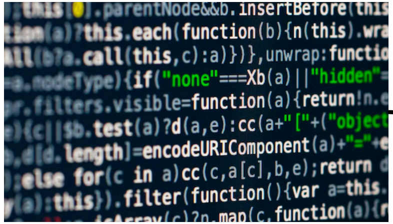

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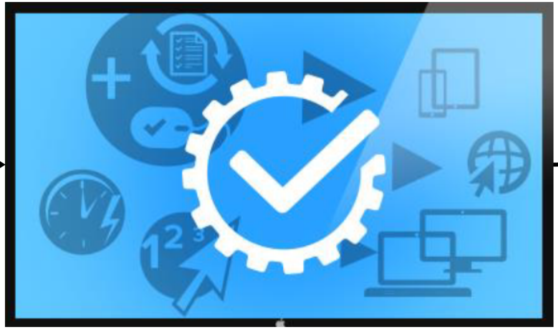

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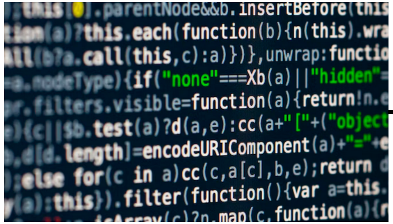

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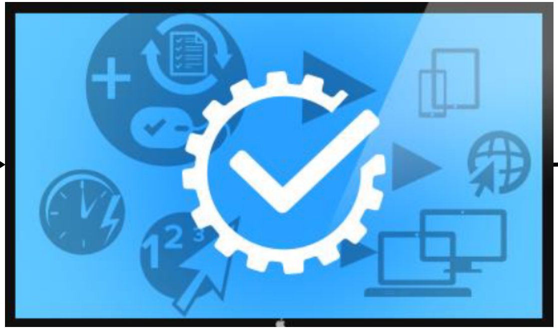

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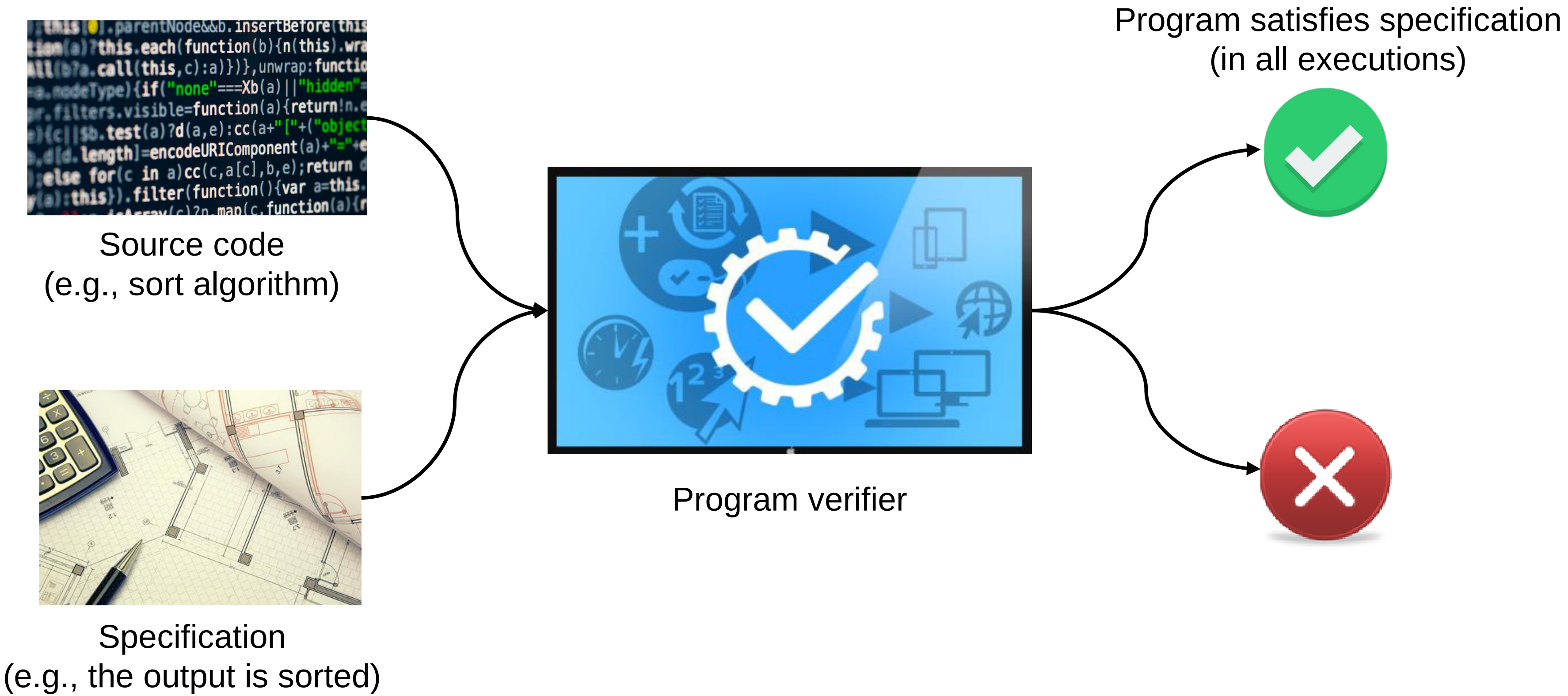

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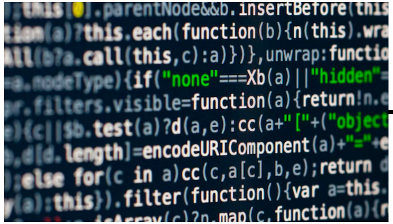

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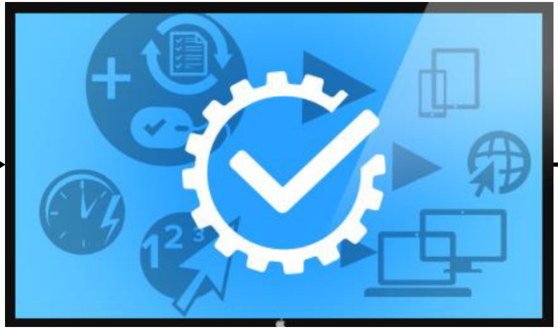

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Program **might** violate specification
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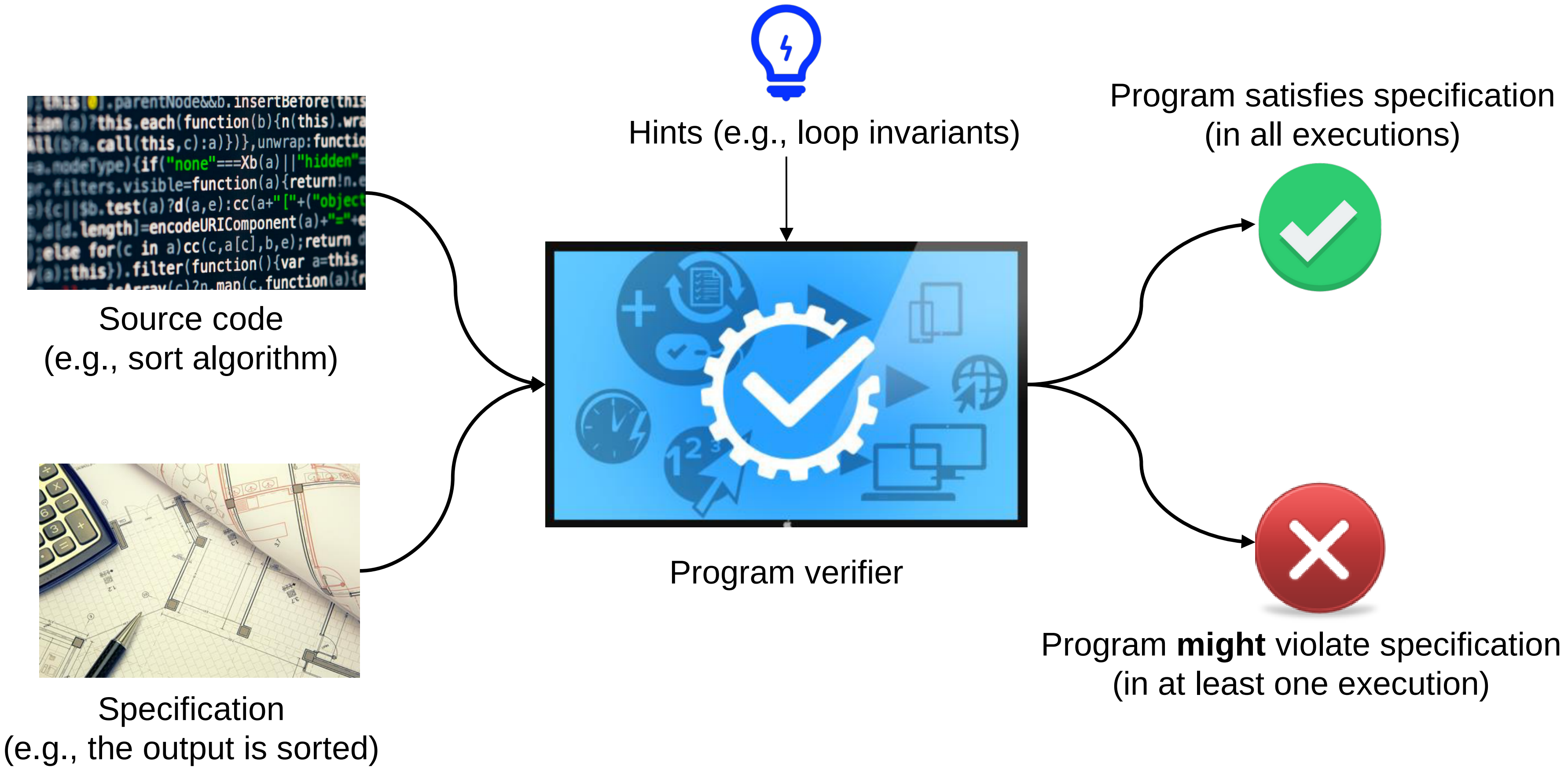

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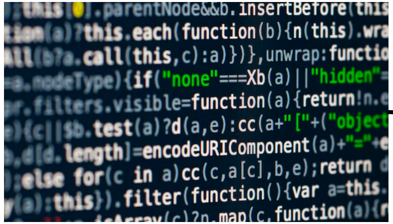

Source code
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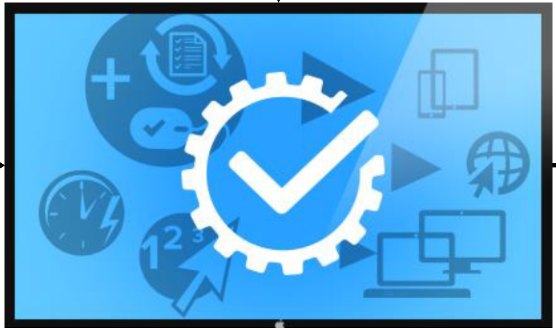
Can include **security** properties
(e.g., information flow security)

Hints (e.g., loop invariants)



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Secure Information Flow: Value Channel

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def compute(h: int, l: int):
  if h > 0:
    res = 1
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Secure Information Flow: Value Channel

high-sensitivity (secret)

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Does *res* leak information about *h*?

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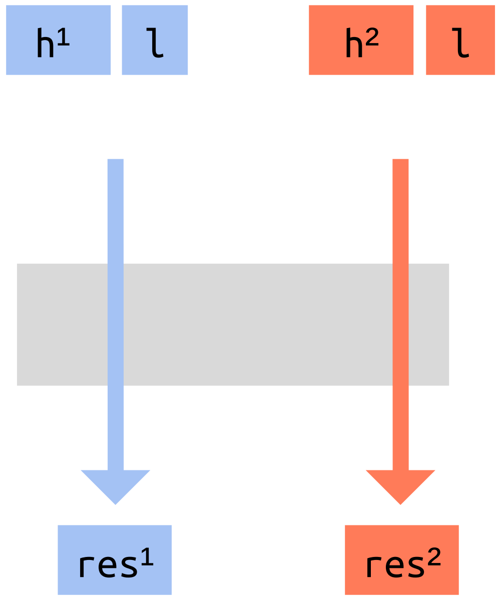

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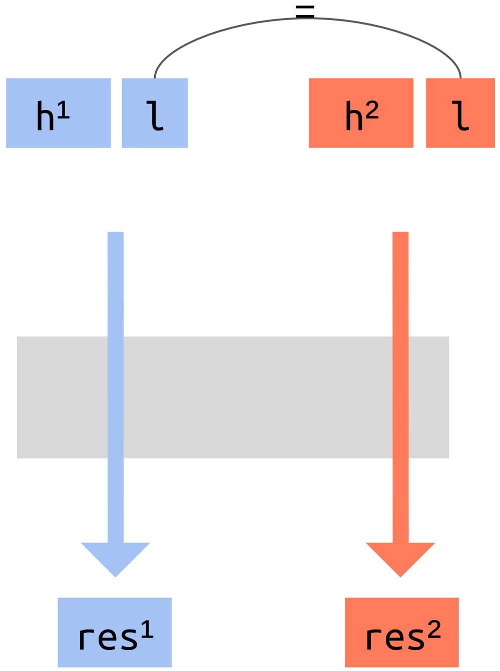

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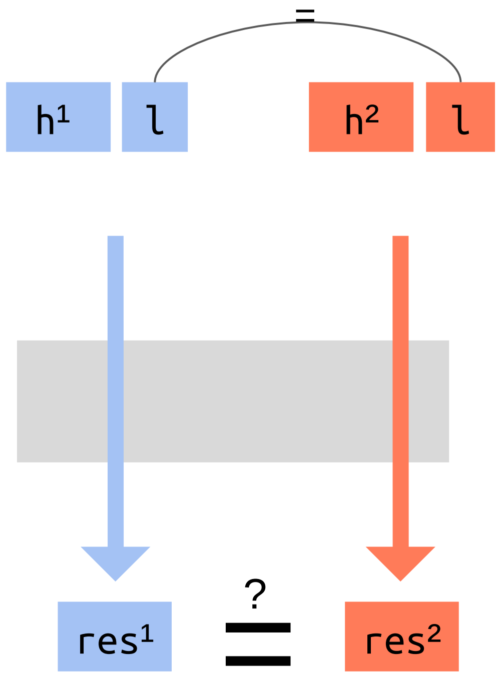

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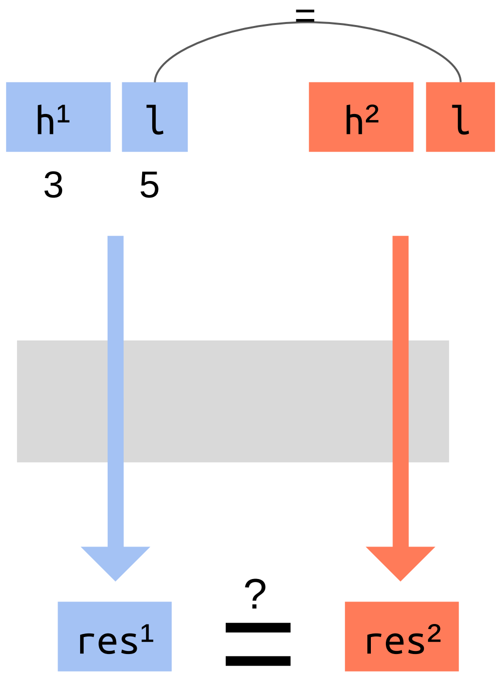

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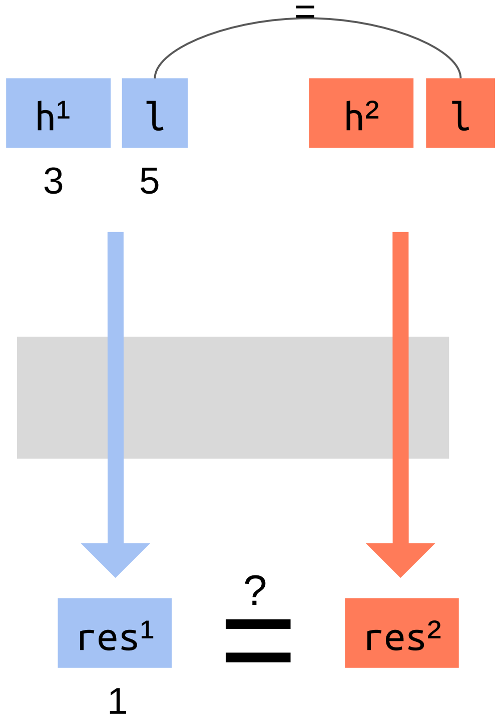

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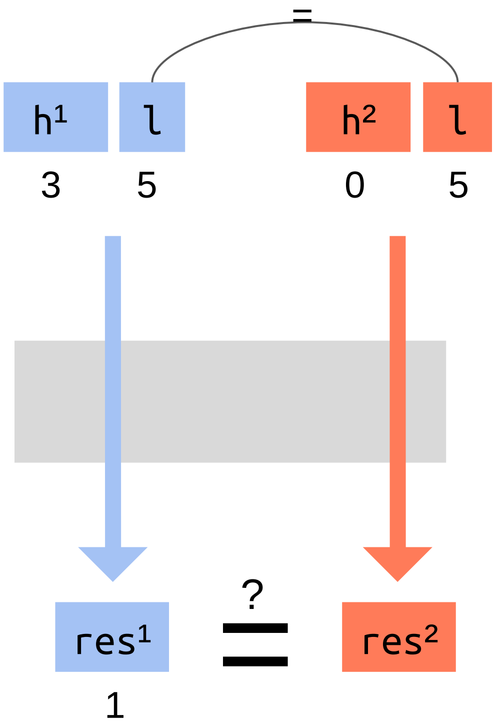

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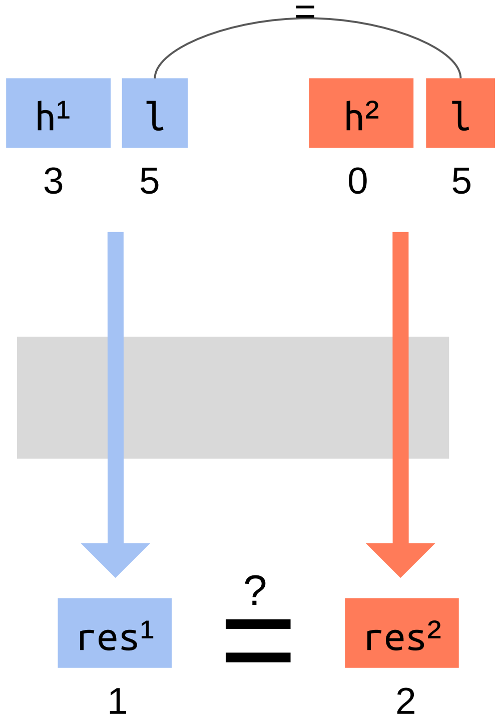

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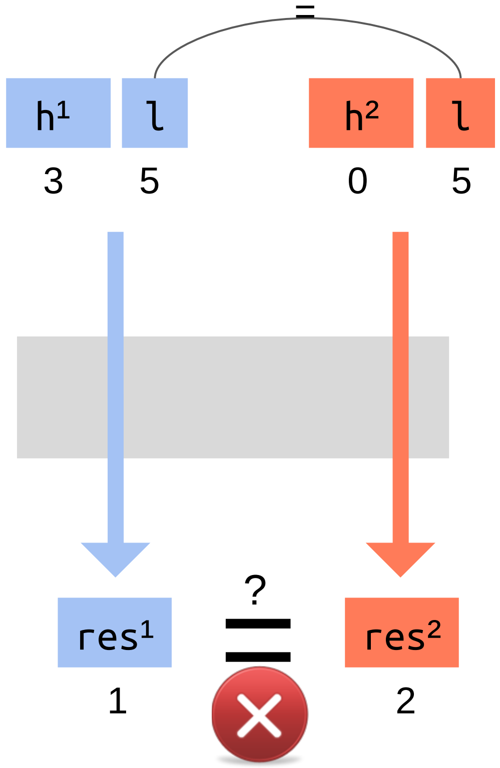

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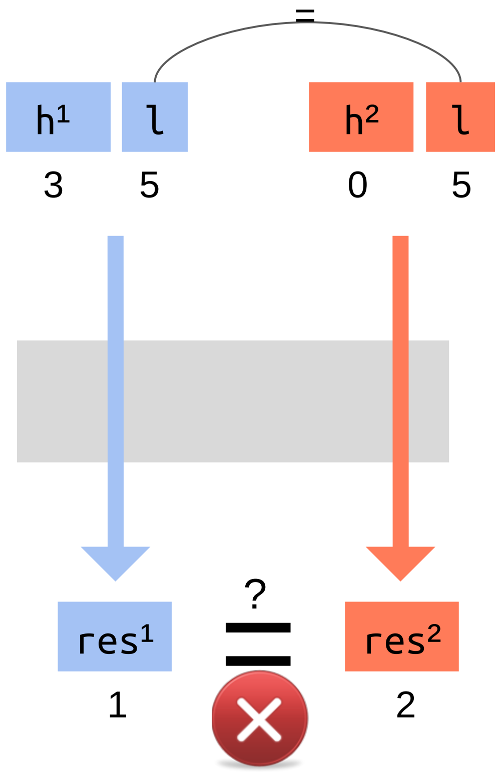

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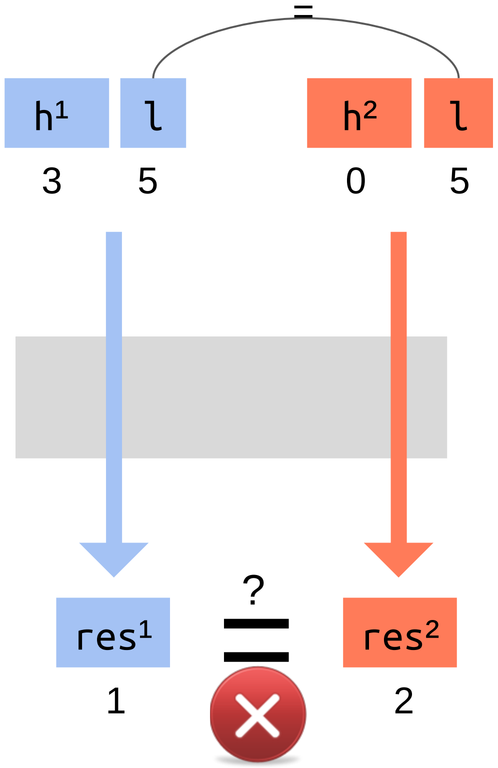

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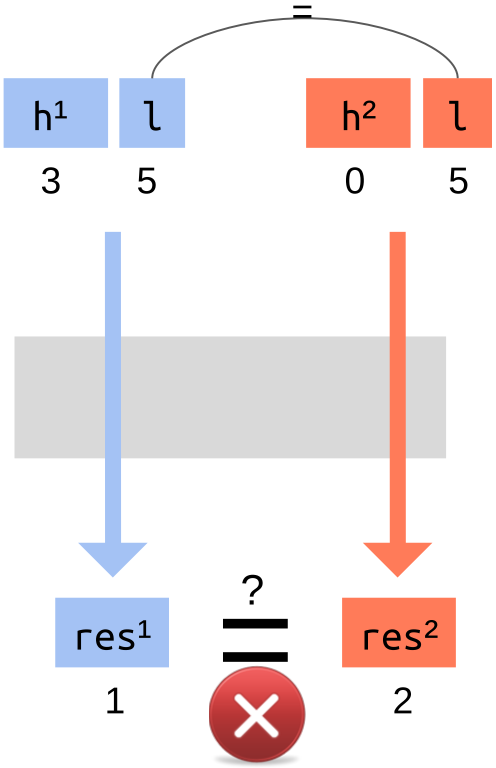

Secure Information Flow: Timing Channel

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```python
def compute(h: int, l: int):
  res = 0
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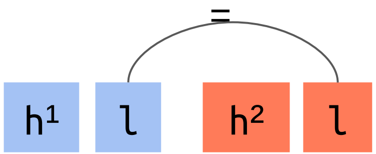
Does the execution time leak information about *h*?

Secure Information Flow: Timing Channel

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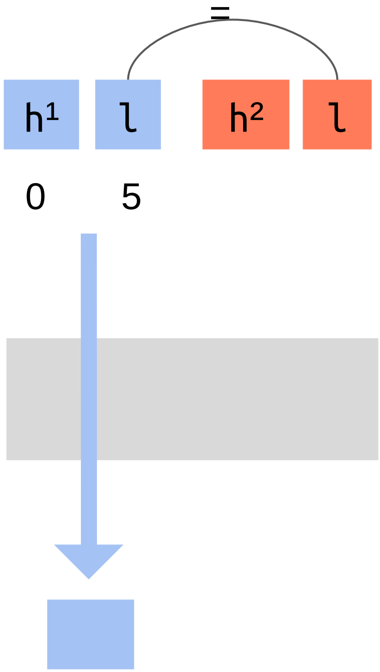

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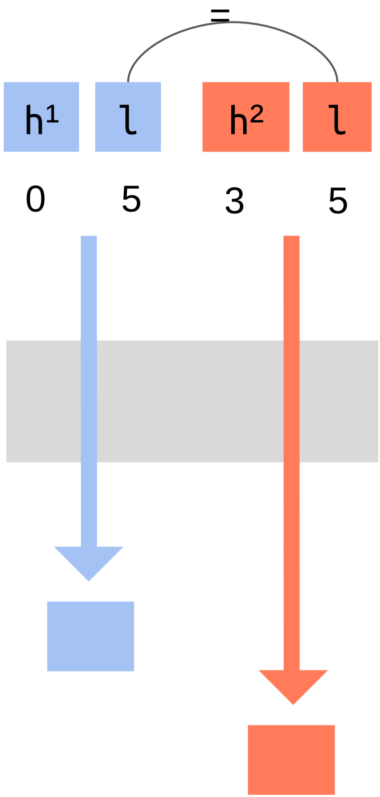

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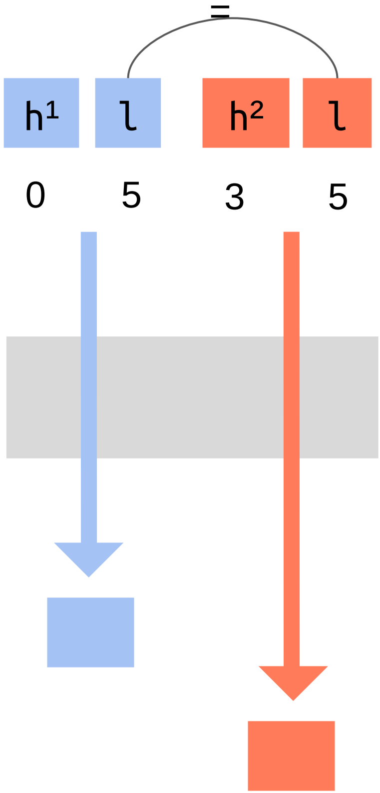

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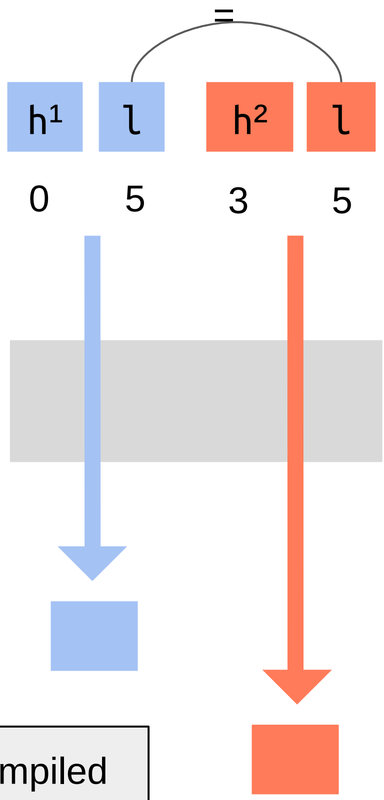


```
          section   .text
_start:   mov       rax, 1
          mov       rdi, 1
          mov       rsi, message
          syscall
          ...
```

The execution time of the compiled program typically depends on values

Secure Information Flow

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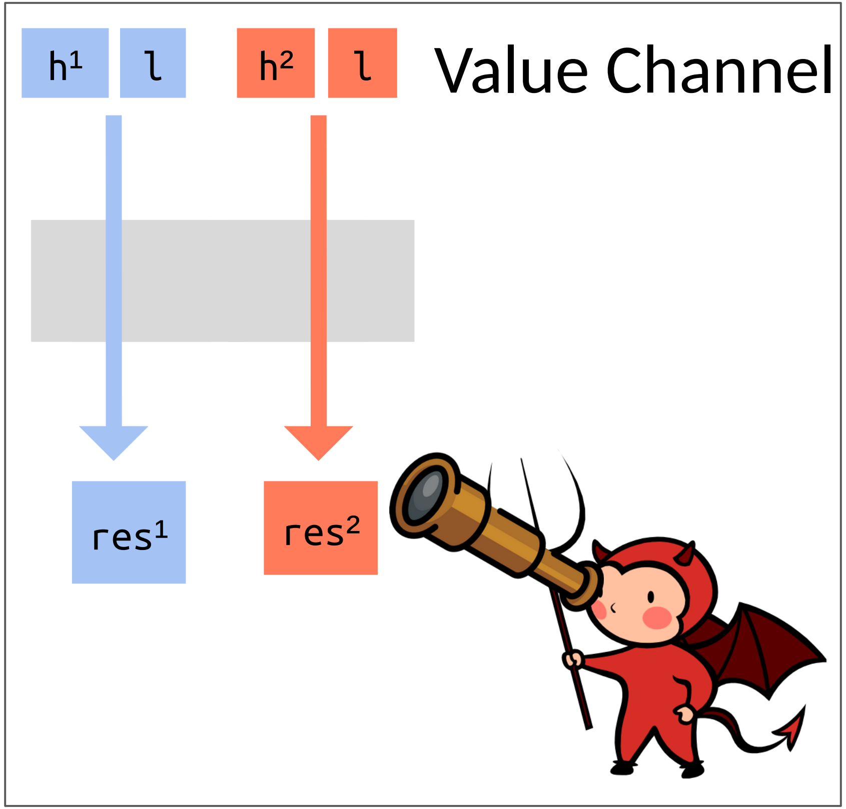

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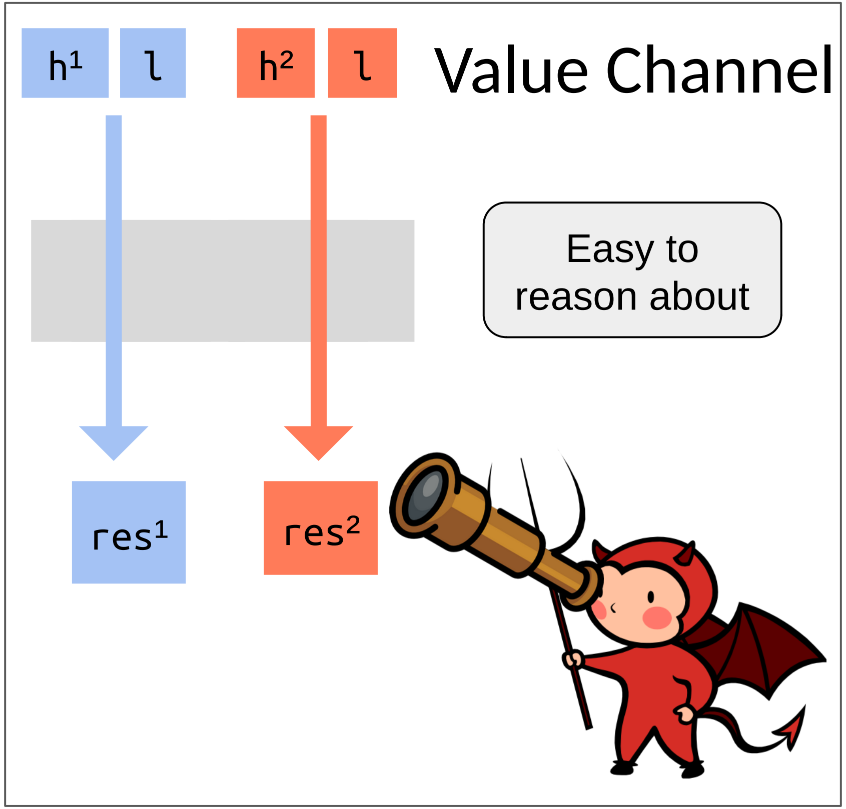

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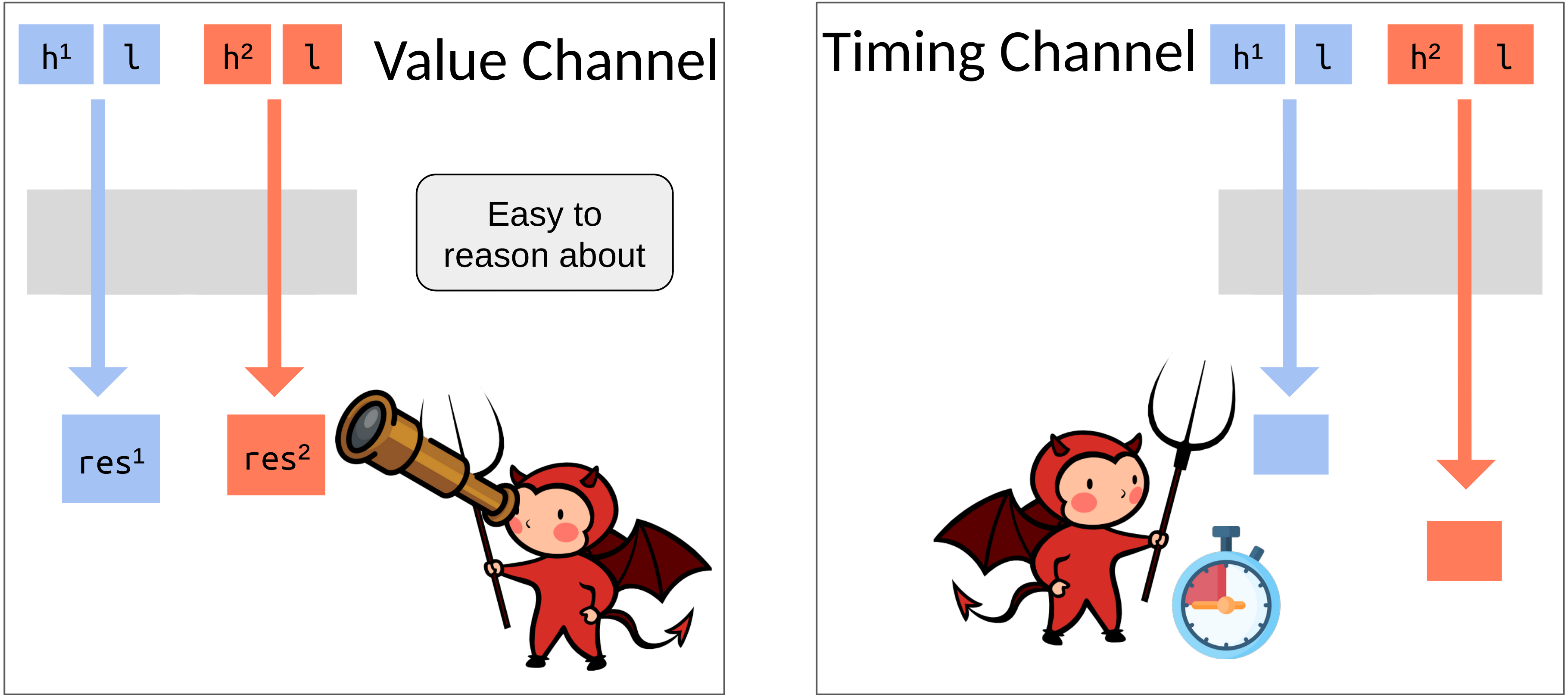

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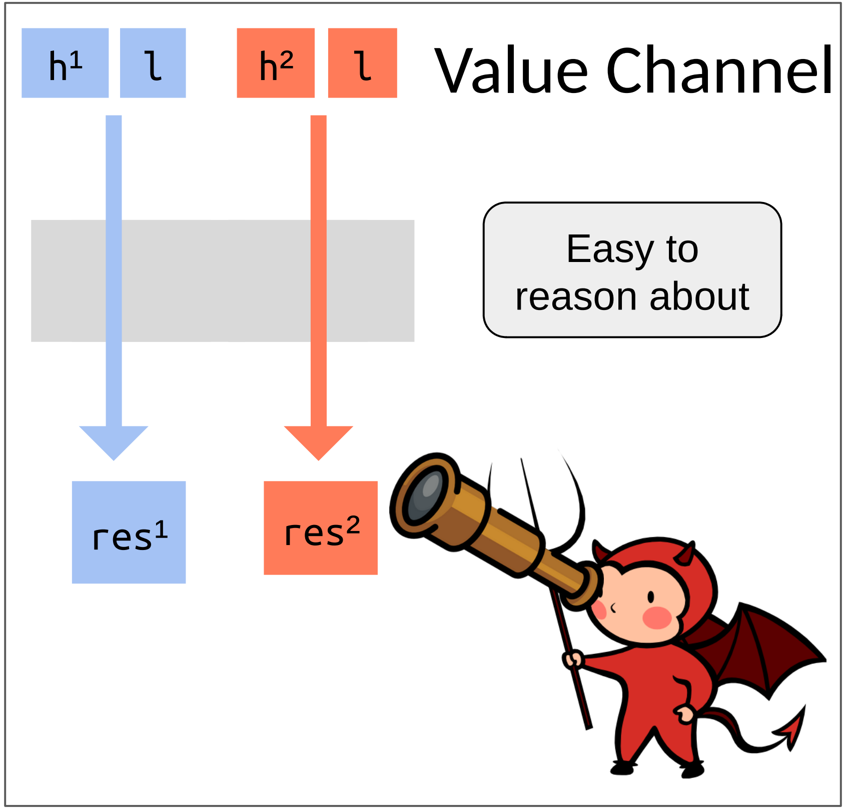

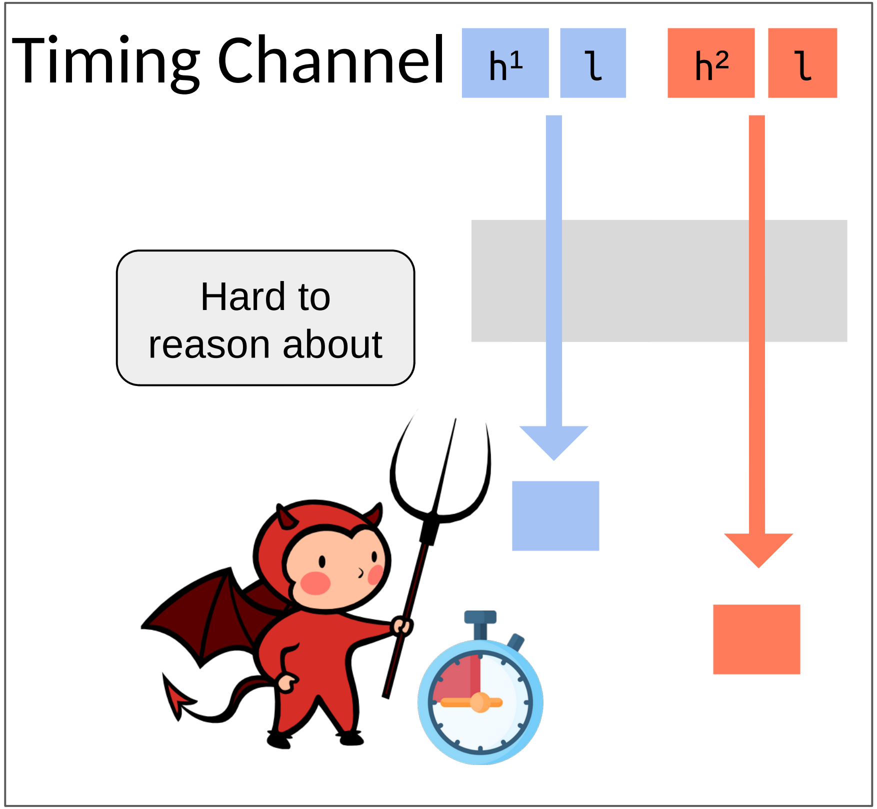

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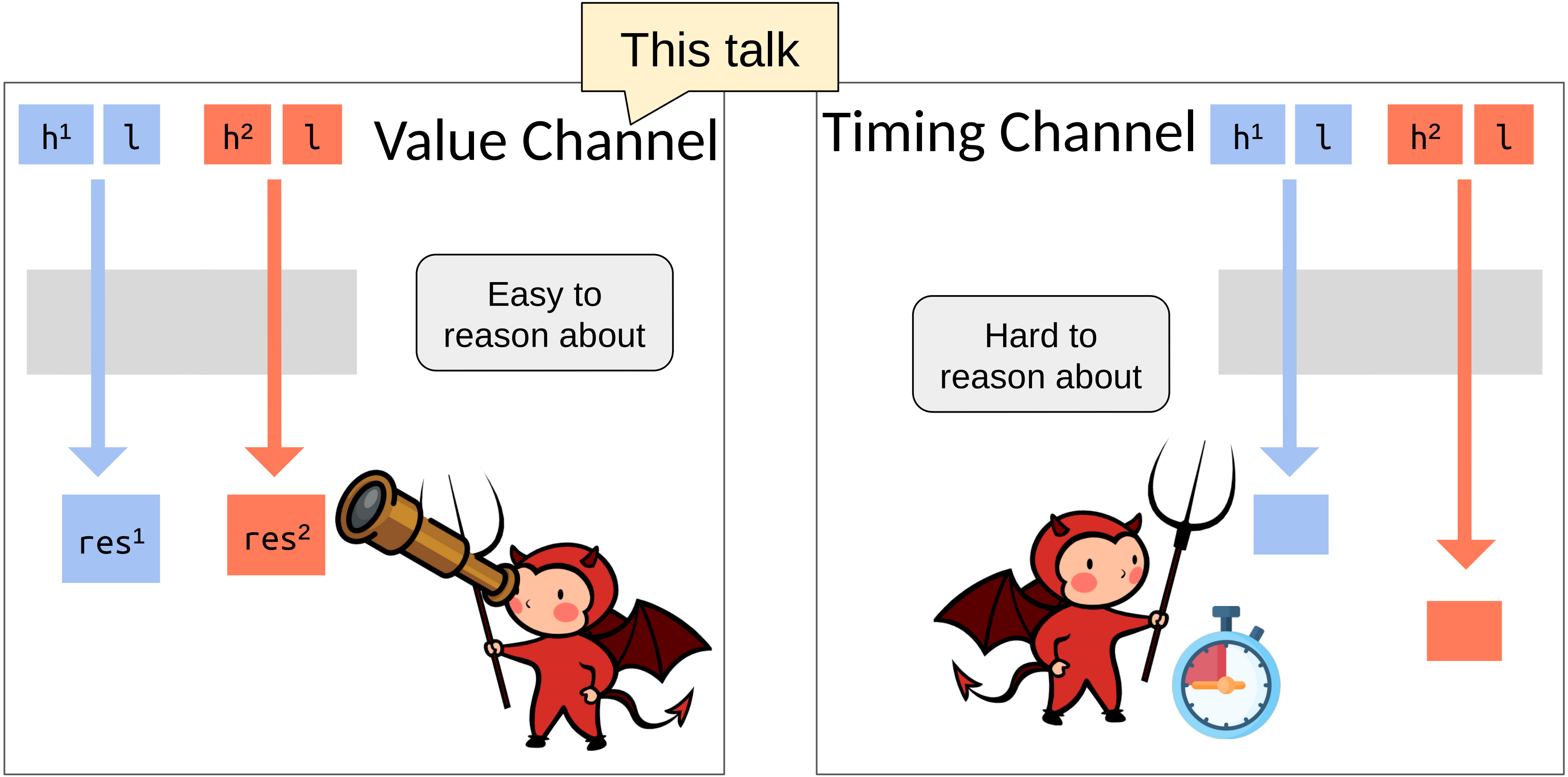

Attacker:
Observes **final results**,
not intermediate state or **timing**

Shared-Memory Concurrency Ruins Everything

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```
while i < h:        ||  while j < 100:
  i += 1            ||    j += 1
shared = 6          ||  shared = 7

        return shared
```

Shared-Memory Concurrency Ruins Everything

Secret-dependent execution time

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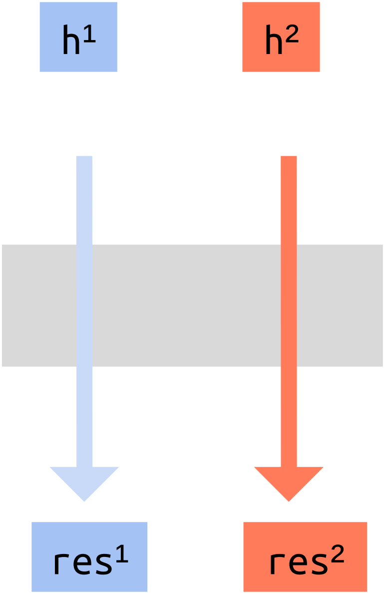

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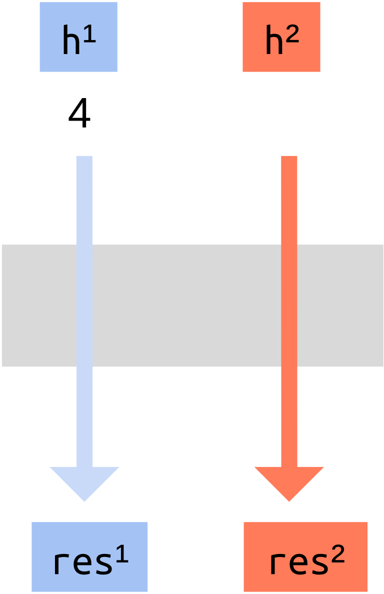

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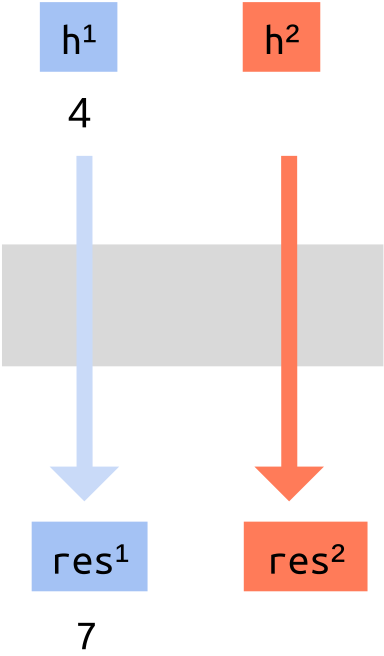

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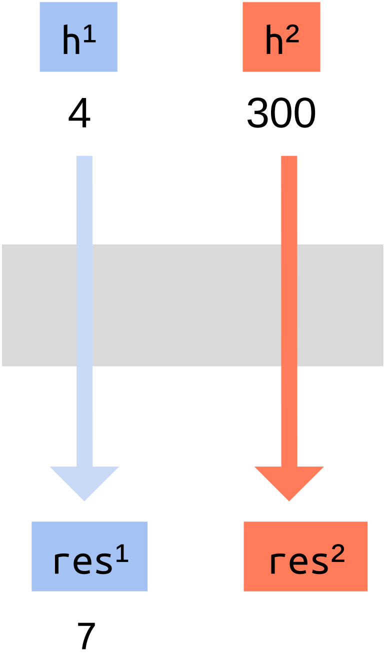

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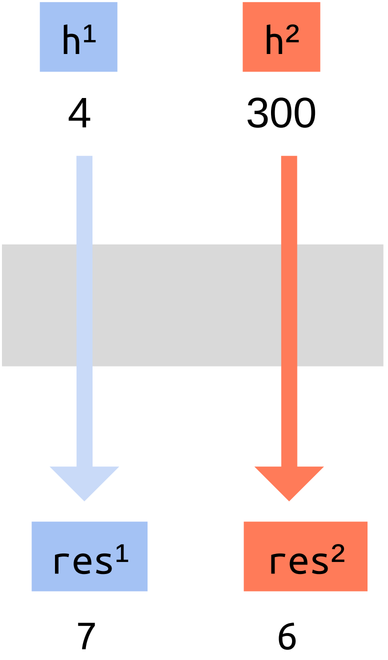

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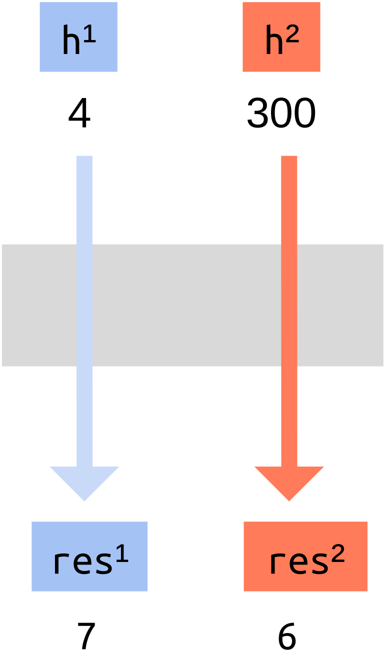

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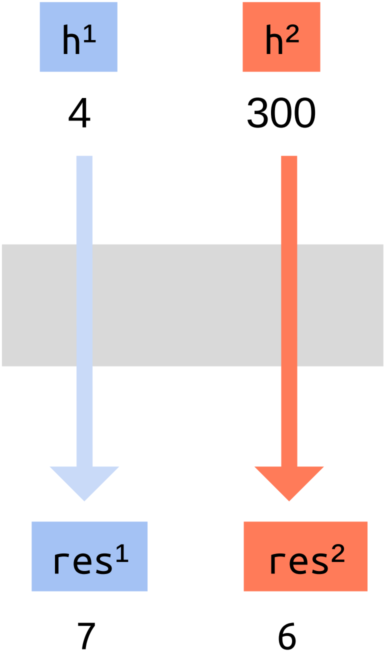


Reasoning about Value Channel

Easy

Reasoning about Value Channel

Easy

Reasoning about Value Channel + Concurrency

Easy

Reasoning about Value Channel + Concurrency

⇓

Reasoning about Timing Channel

Easy

Reasoning about Value Channel + Concurrency

Reasoning about Timing Channel

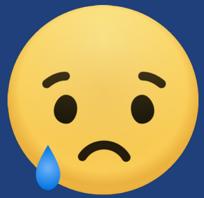
Hard

Easy

Reasoning about Value Channel + Concurrency

Reasoning about Timing Channel

Hard



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‖

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Secret value

↓ influences

Execution time

Shared-Memory Concurrency Ruins Everything

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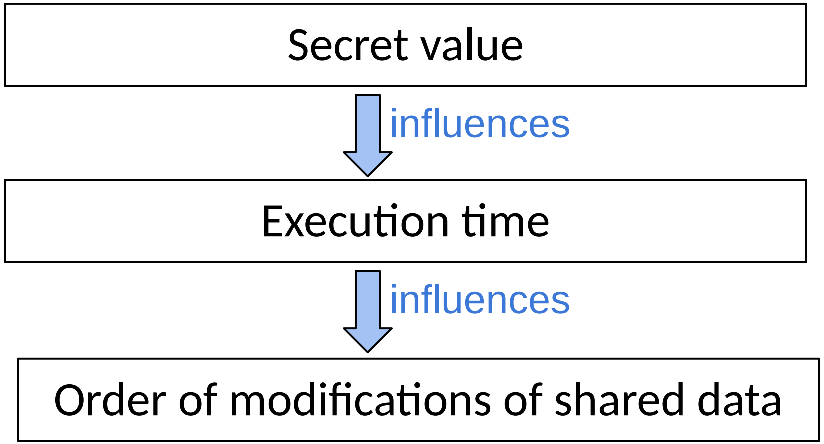
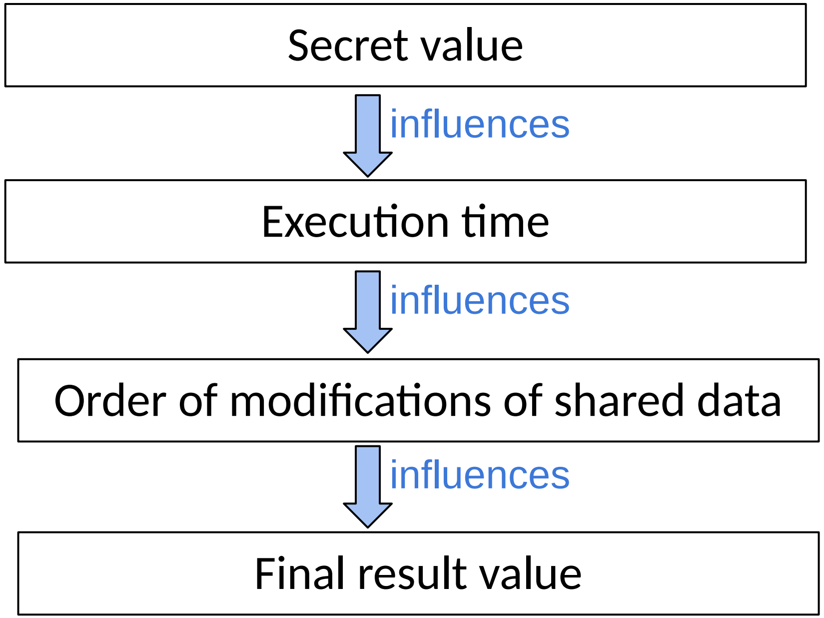


Shared-Memory Concurrency Ruins Everything

```
while i < h:          ||   while j < 100:
  i += 1              ||     j += 1
shared = 6            ||   shared = 7

               return shared
```



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while i < h:          ||   while j < 100:
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shared = 6            ||   shared = 7

                return shared
```

Secret value

↓ influences

Execution time

↓ influences

Order of modifications of shared data

↓ influences

Final result value

Existing (Modular) Solutions

```
while i < h:          ||  while j < 100:
  i += 1              ||    j += 1
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```

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```
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shared = 6           ||   shared = 7

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```

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~~in ences~~

Execution time

influences

Order of modifications of shared data

influences

Final result value

Existing (Modular) Solutions

Insecure

```
while i < h:          ||    while j < 100:
  i += 1                      j += 1
shared = 6                  shared = 7

                  return shared
```

Secret value

~~influences~~

Execution time

influences

Order of modifications of shared data

influences

Final result value

Existing (Modular) Solutions

Insecure

```
while i < h:          ||   while j < 100:
  i += 1              ||     j += 1
shared = 6            ||   shared = 7

          return shared
```

```
          shared = l

while i < h:          ||   while j < 100:
  i += 1              ||     j += 1
atomic:               ||   atomic:
  shared += 6         ||     shared += 7

          return shared
```

Secret value

↓ ~~influences~~

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Existing (Modular) Solutions

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```
while i < h:          ||   while j < 100:
  i += 1              ||     j += 1
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              shared = l
while i < h:          ||   while j < 100:
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atomic:               ||   atomic:
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Execution time

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Order of modifications of shared data

influences

Final result value

Problem Statement

Reason about **values** in concurrent programs
without reasoning about **timing**
and without considering all **interleavings**

Key Idea

Order does not influence result if modifications **commute**

Our Solution: Commutativity

Secret values

↓ influence

Execution time

↓ influences

Order of modifications of shared data

↓ influences

Final result value

Our Solution: Commutativity

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influence

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influences

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~~influences~~

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```
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while i < h:              ||   while j < 100:
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```

Secret values

↓ influence

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Order of modifications of shared data

↓ ~~influences~~

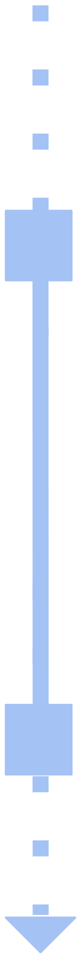
Final result value

Basic Solution

```
             shared = …
atomic:                 ||  atomic:
  shared = A            ||    shared = B
atomic:                 ||
  shared = C            ||
                  ...
```

Basic Solution

```
shared = …
atomic:            ||  atomic:
  shared = A       ||    shared = B
atomic:            ||
  shared = C       ||
                 …
```



Basic Solution

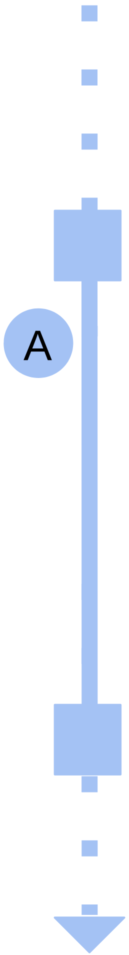
shared = …

```
atomic:
  shared = A
atomic:
  shared = C
```

||

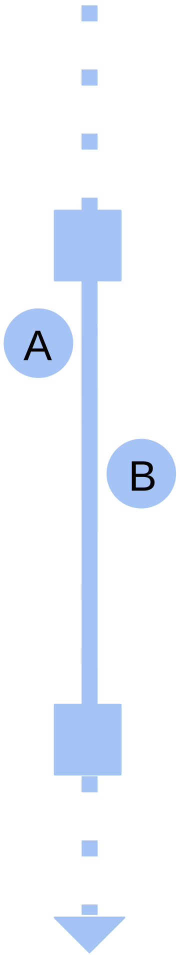
```
atomic:
  shared = B
```

…



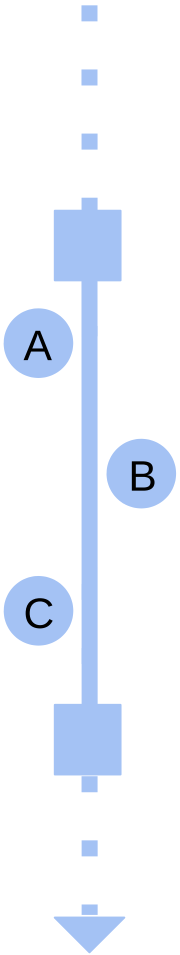
Basic Solution

```
        shared = …
atomic:              ||  atomic:
   shared = A        ||     shared = B
atomic:              ||
   shared = C        ||
           …
```



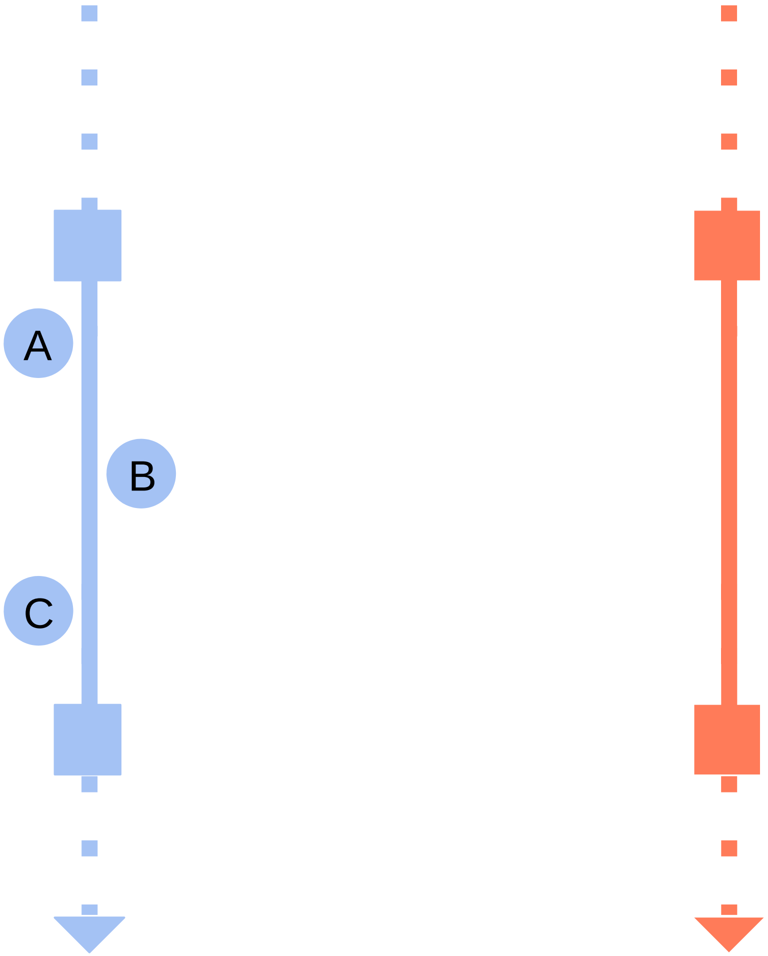
Basic Solution

```
shared = …
atomic:              ||  atomic:
  shared = A         ||    shared = B
atomic:              ||
  shared = C         ||
          …
```



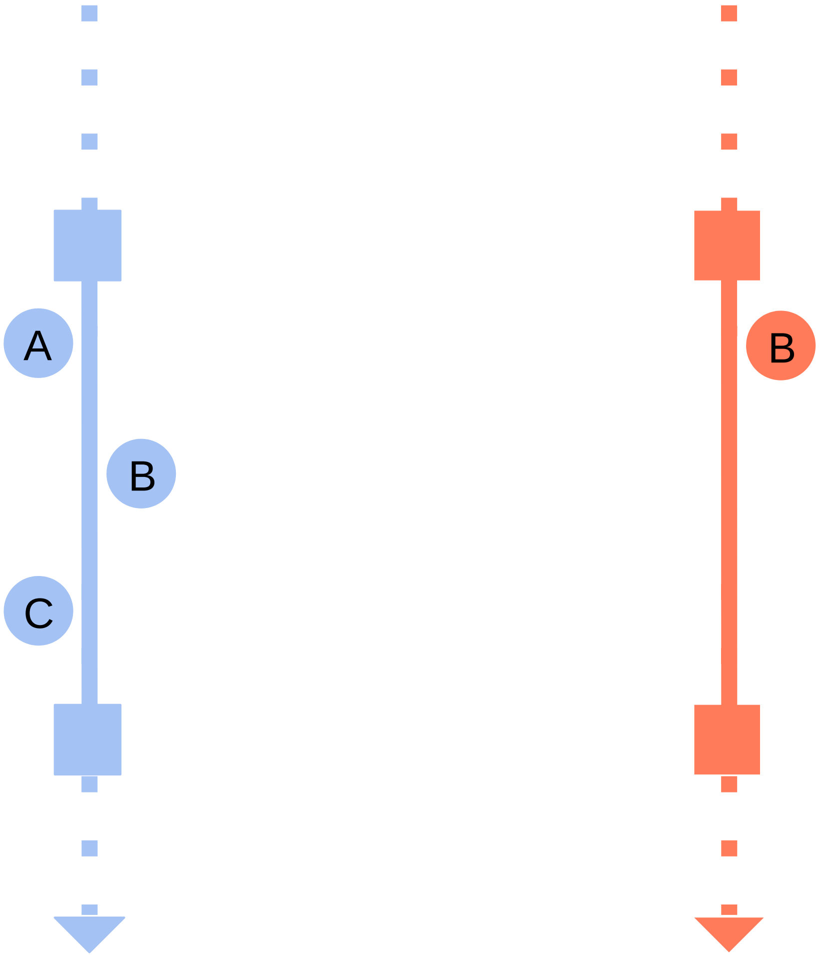
Basic Solution

```
shared = …
atomic:            ||  atomic:
  shared = A       ||    shared = B
atomic:            ||
  shared = C       ||
                   …
```



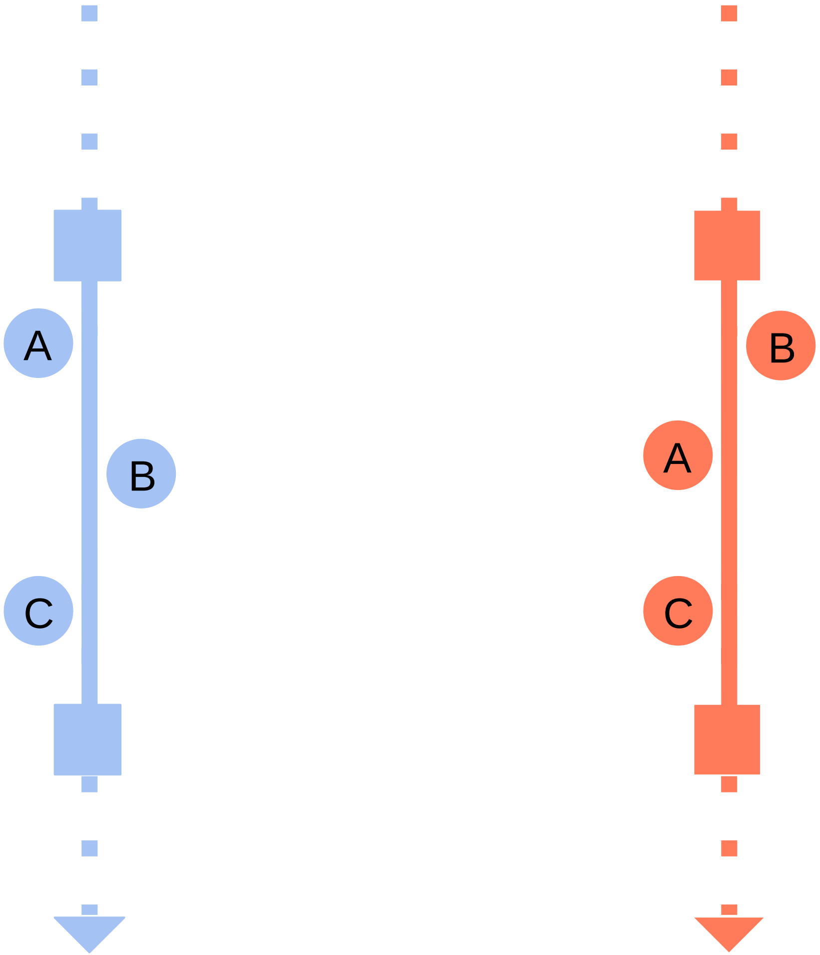
Basic Solution

```
          shared = …
atomic:                  ||  atomic:
   shared = A            ||     shared = B
atomic:                  ||
   shared = C            ||
               …
```



Basic Solution

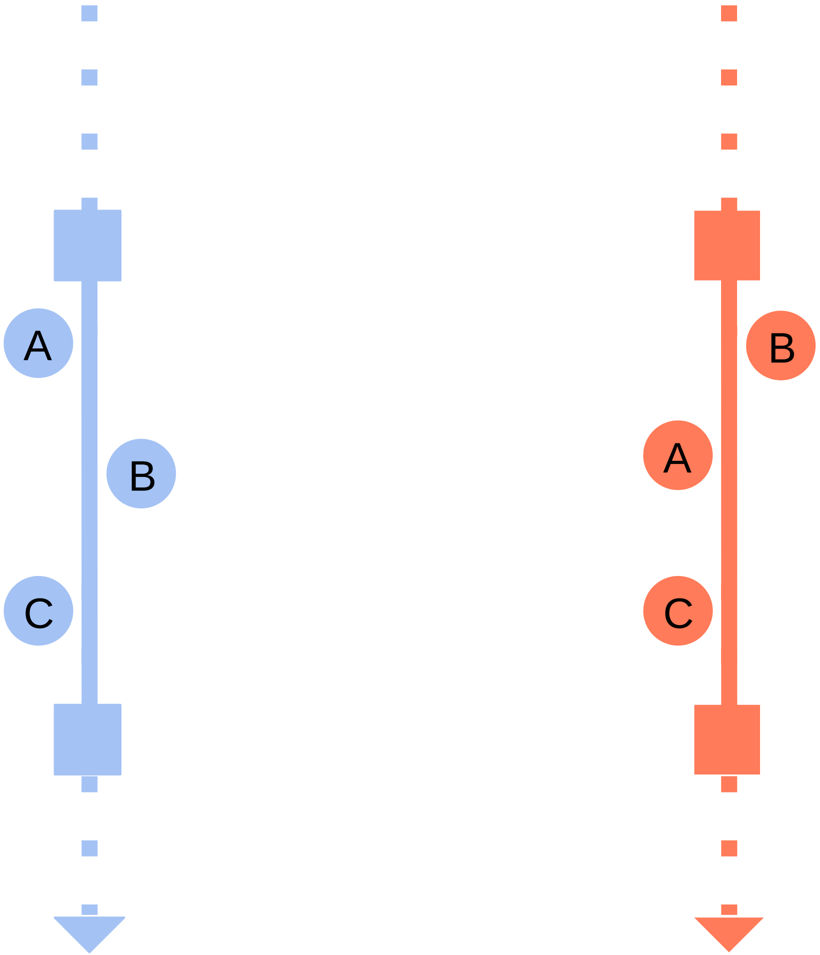
```
shared = …
atomic:              ||  atomic:
  shared = A         ||    shared = B
atomic:              ||
  shared = C         ||
            …
```



Basic Solution

```
shared = …
atomic:            ||  atomic:
   shared = A      ||     shared = B
atomic:            ||
   shared = C      ||
        …
```

If



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            shared = …
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atomic:              ||
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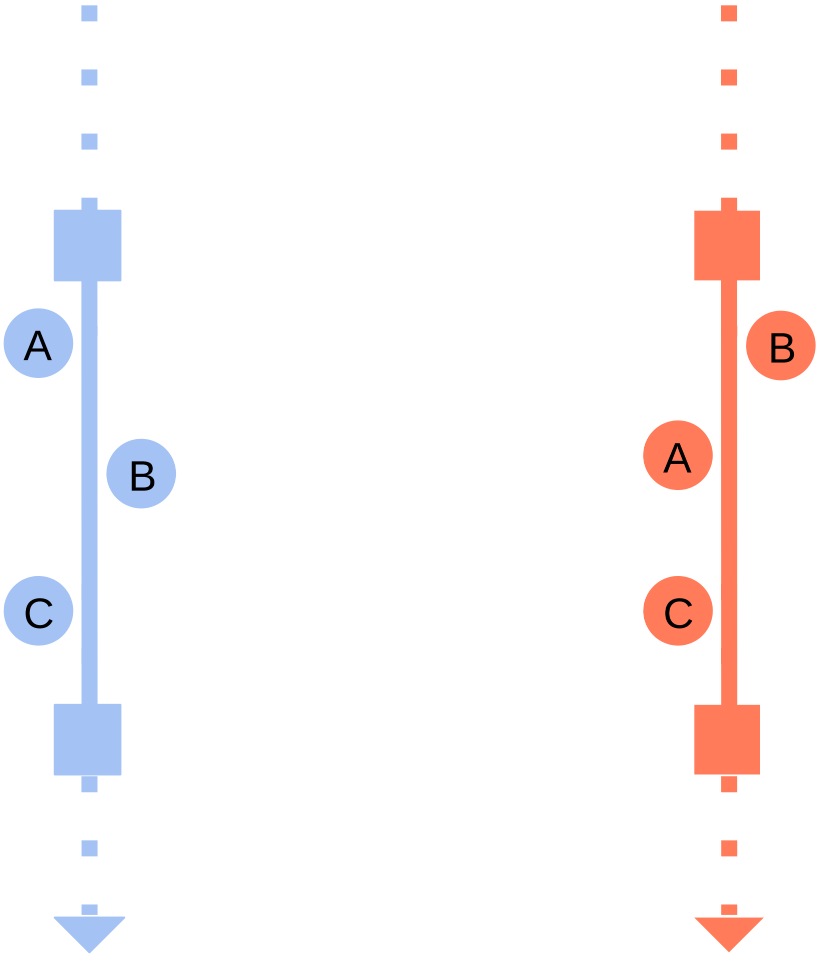

If

(1) *shared* has the same initial value in both executions

Basic Solution

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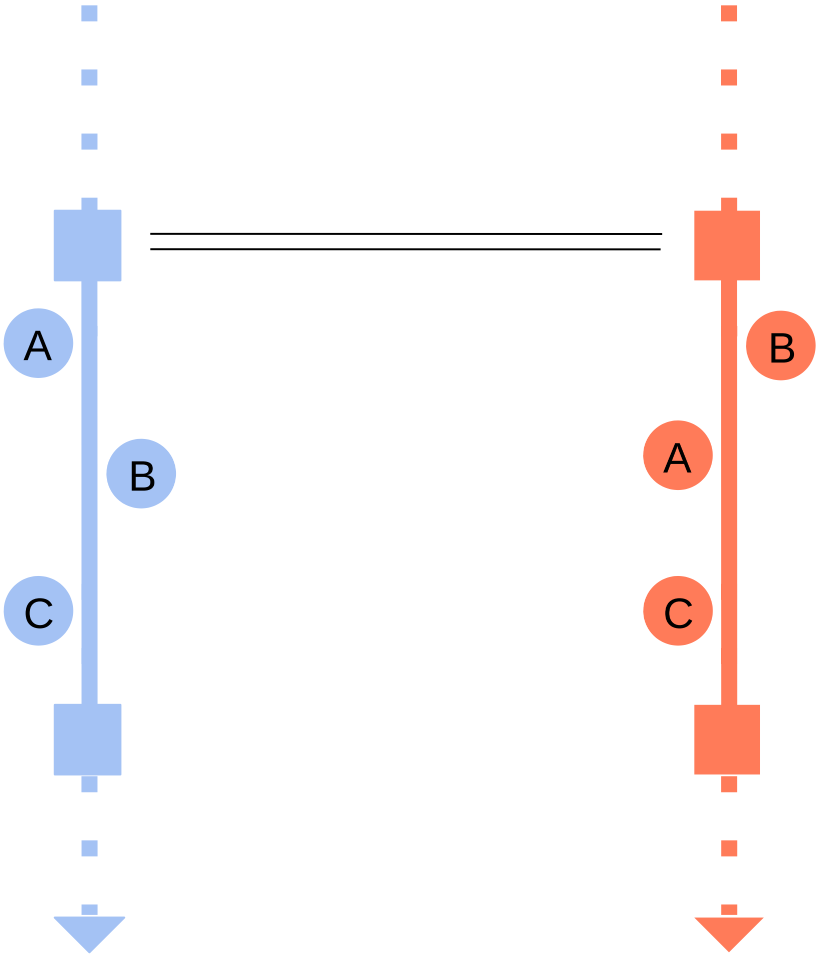
shared = …
atomic:                 ||  atomic:
  shared = A            ||    shared = B
atomic:                 ||
  shared = C            ||
             …
```



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Basic Solution

shared = …

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atomic:                   ||  atomic:
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atomic:                   ||
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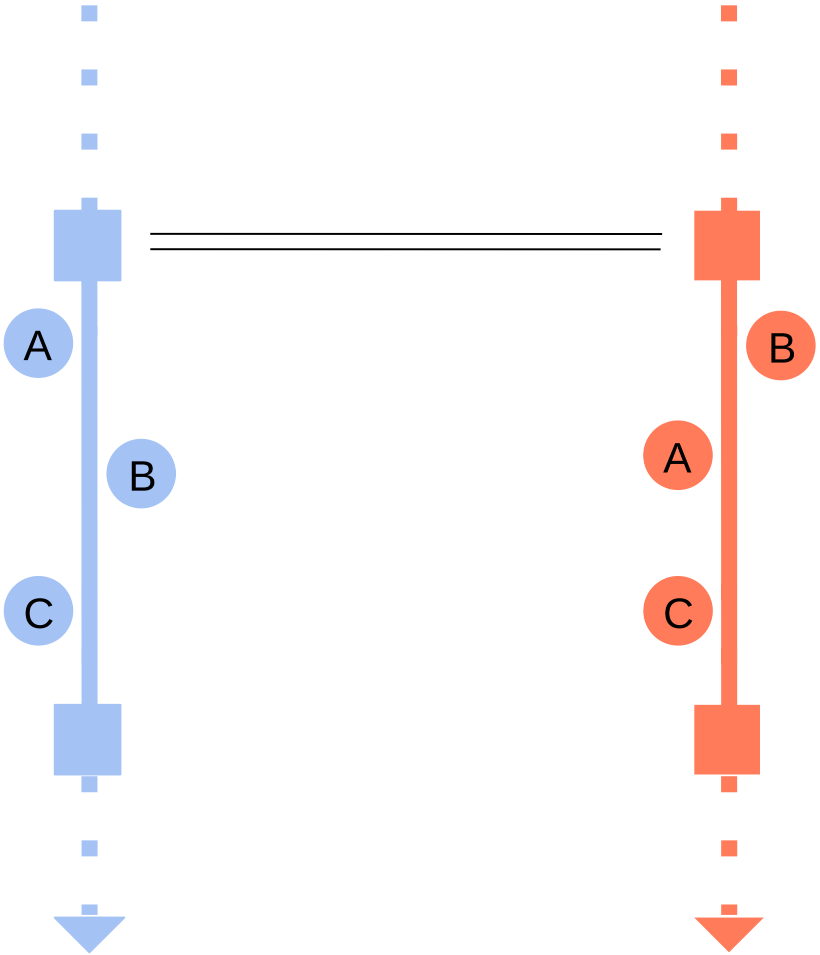
```

…



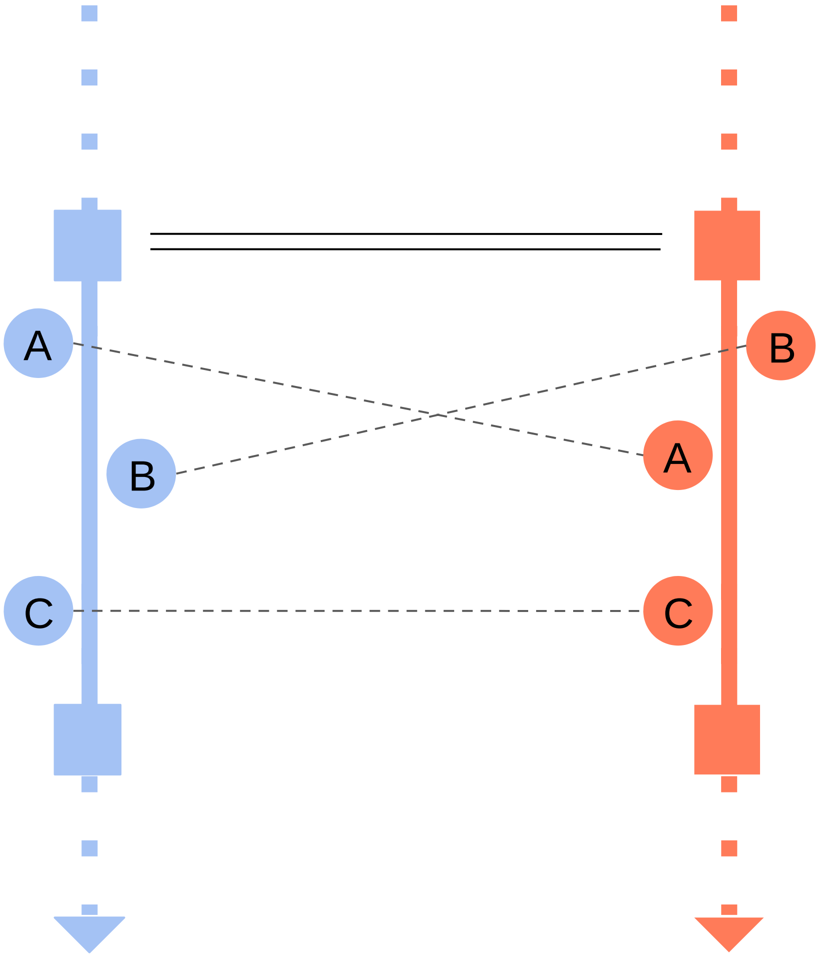
If

(1) *shared* has the same initial value in both executions

(2) the two executions perform the “same” modifications

Basic Solution

```
shared = …
atomic:                 ||  atomic:
  shared = A            ||    shared = B
atomic:                 ||
  shared = C            ||
          …
```



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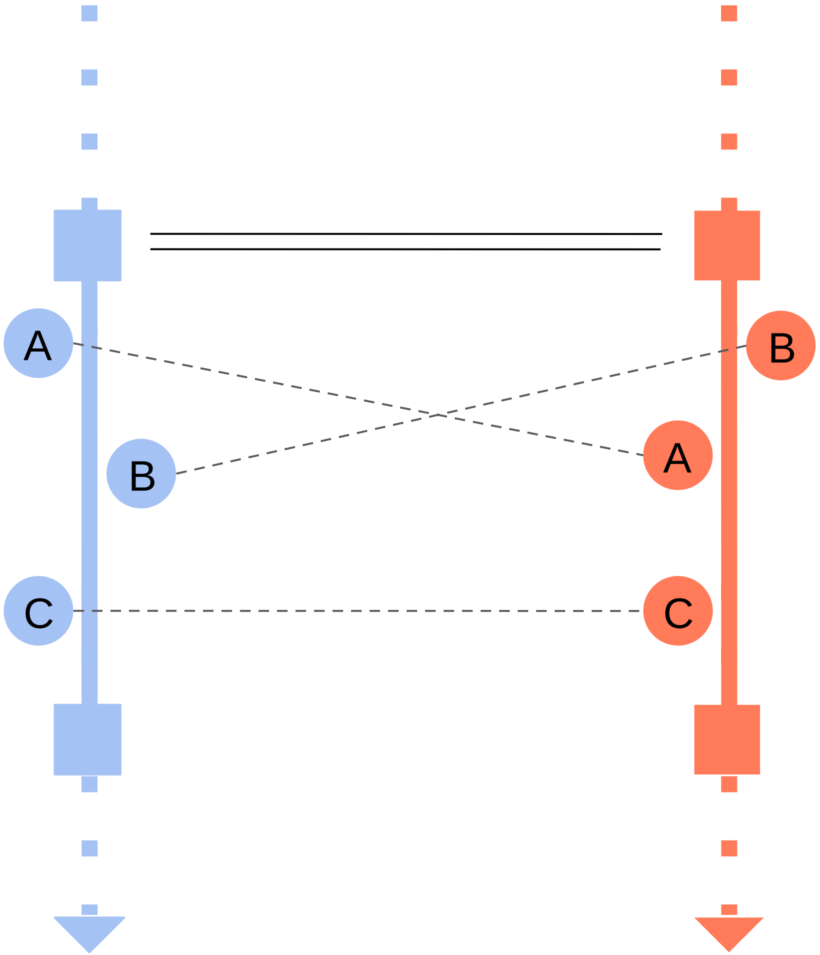

If

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(3) the modifications commute

Basic Solution

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            shared = …
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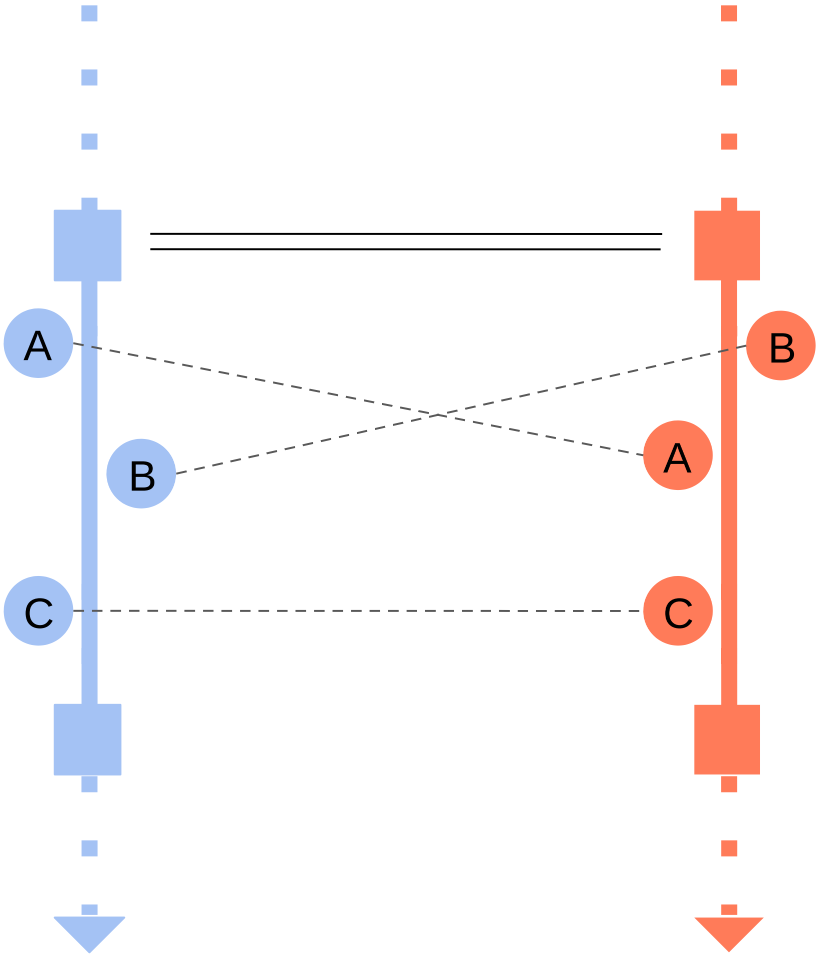

If

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then *shared* has the same final value in both executions

Basic Solution

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   shared = C          ||
                ...
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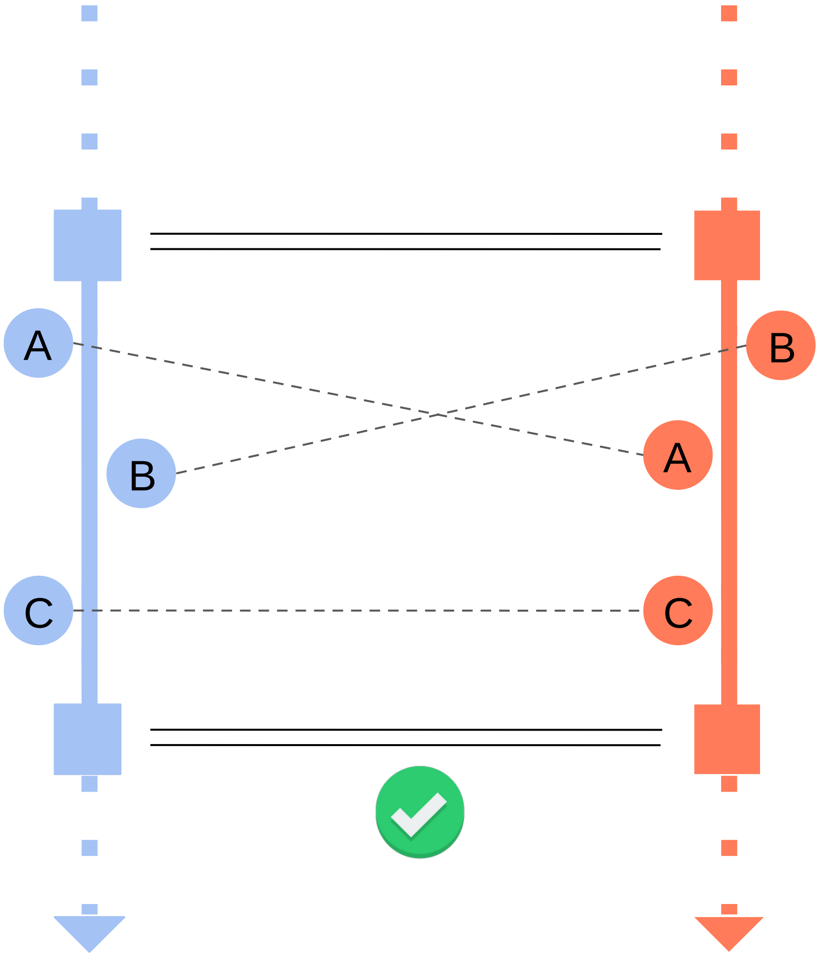

If

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then *shared* has the same final value in both executions

Basic Solution

shared = 0

```
atomic:                 ||  ...
   shared += 1          ||  atomic:
atomic:                 ||     shared += 5
   shared += 3          ||
                 ...
```

If

(1) *shared* has the same initial value in both executions

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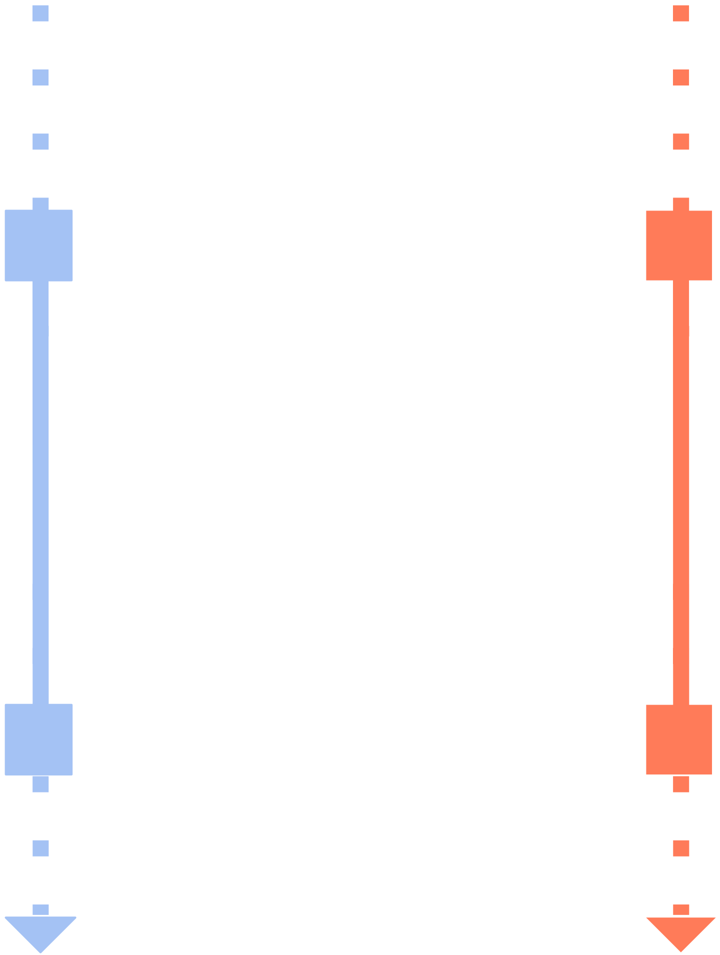

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  shared += 3          ||
                  ...
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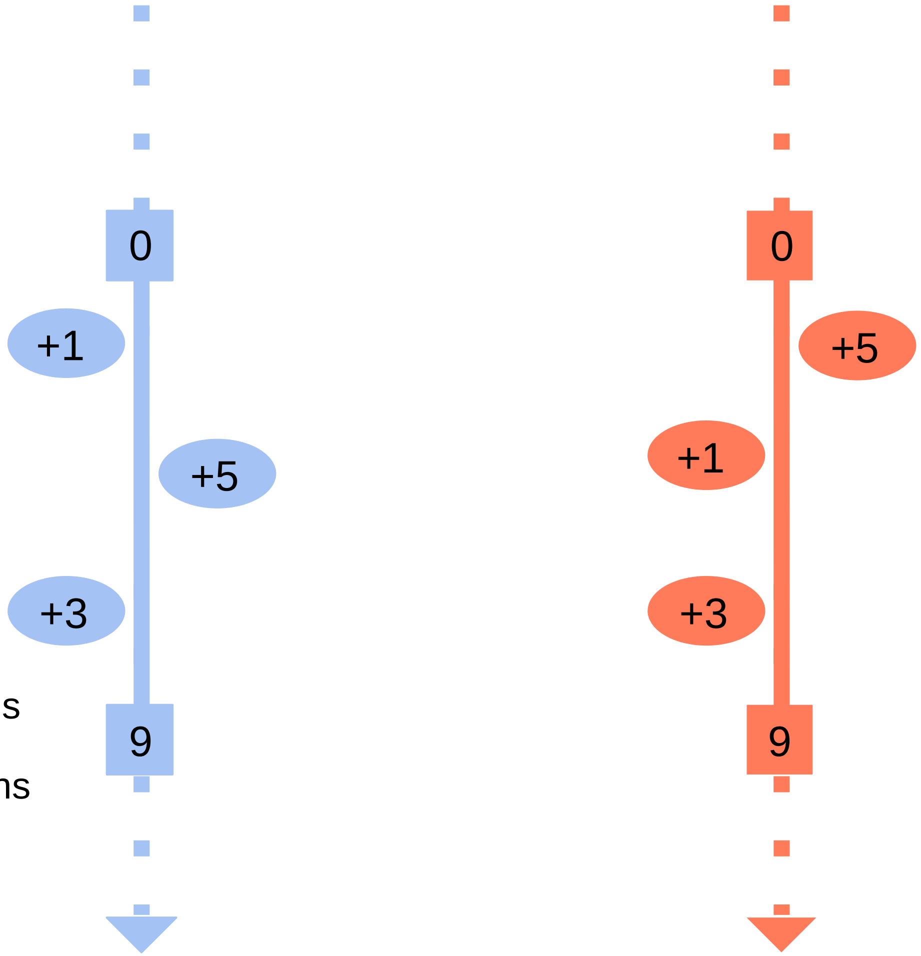

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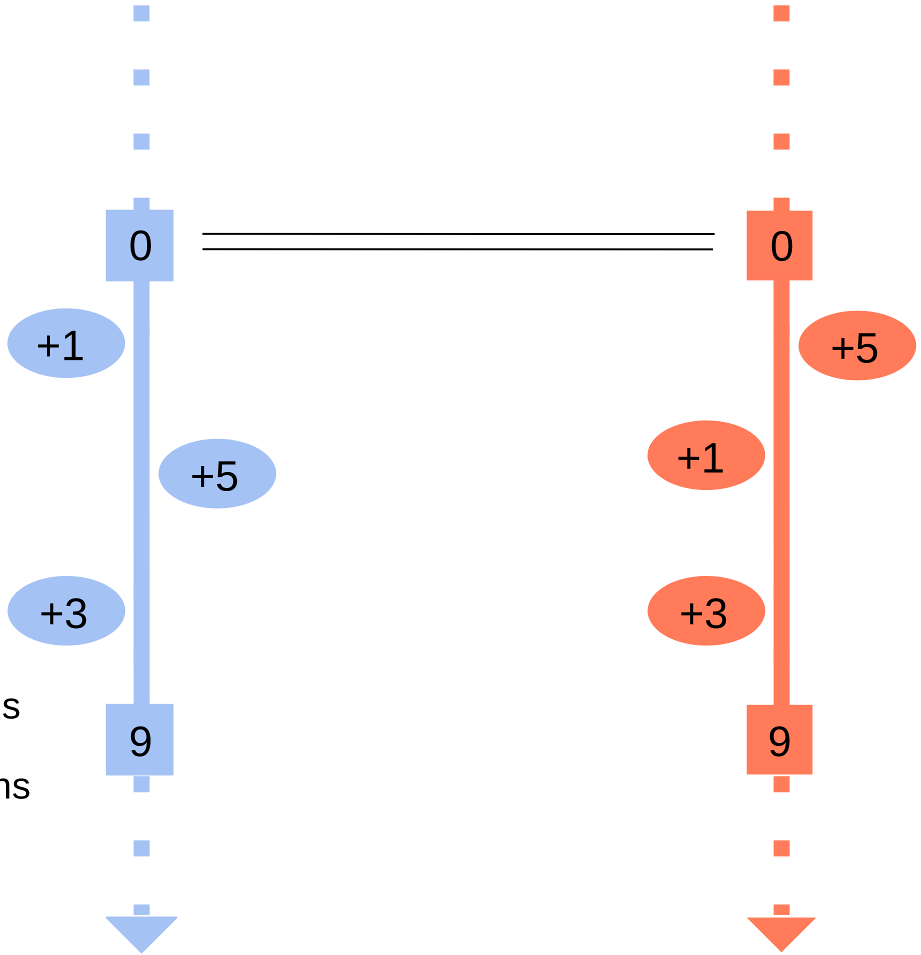

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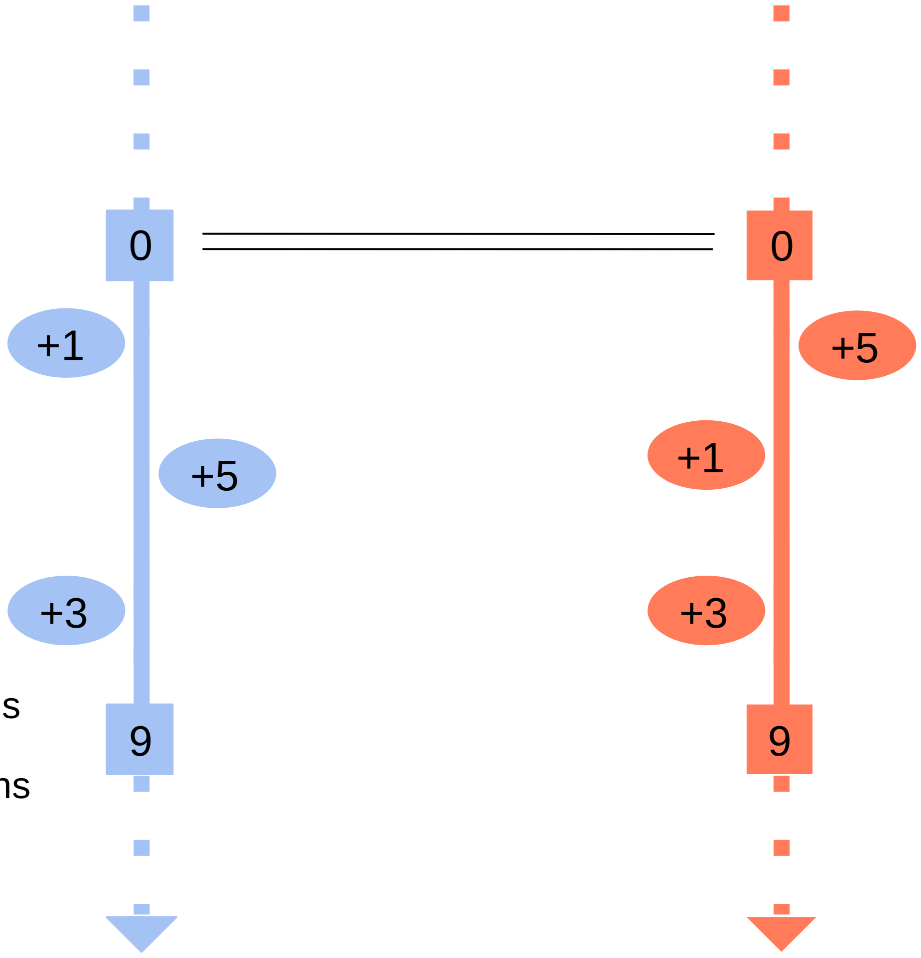

Basic Solution

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            shared = 0
atomic:              ||  ...
  shared += 1        ||  atomic:
atomic:              ||    shared += 5
  shared += 3        ||
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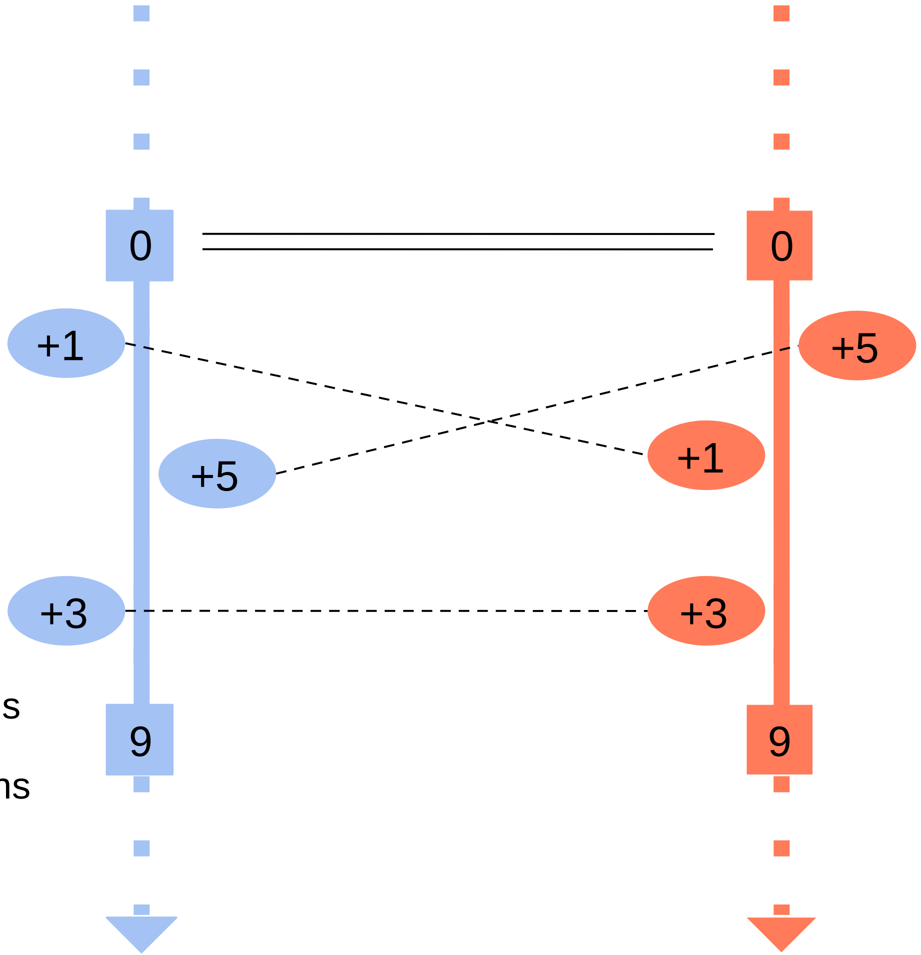

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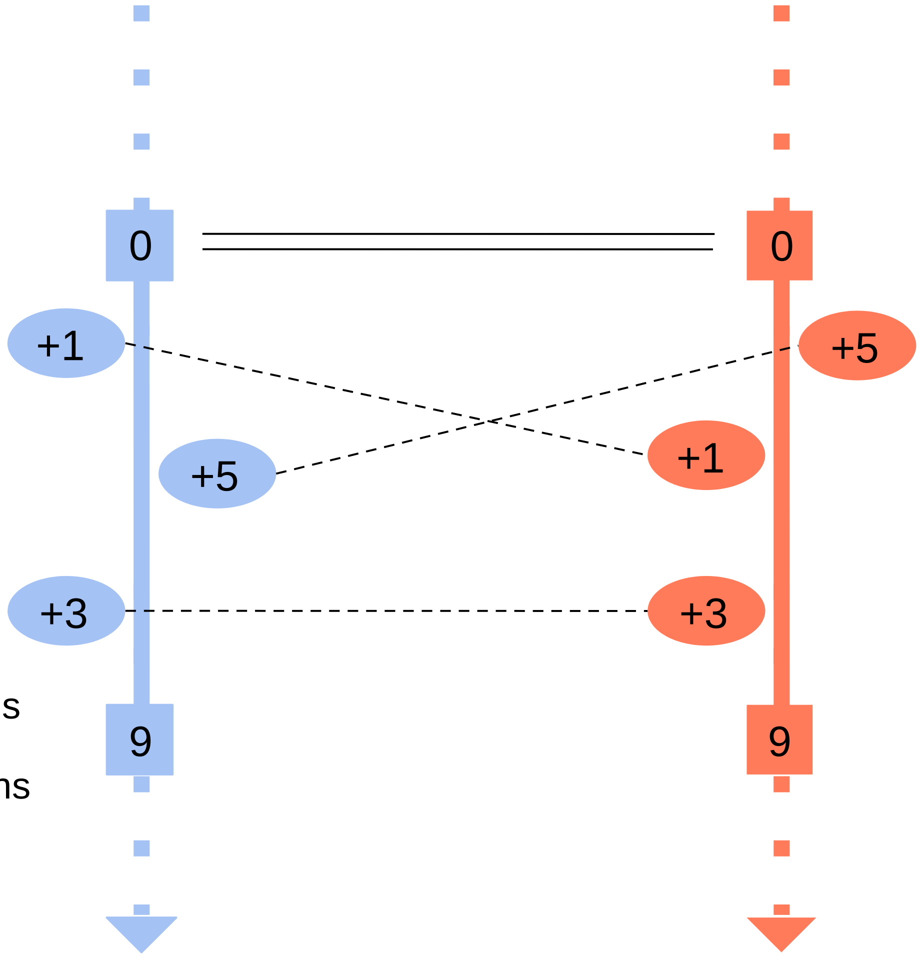

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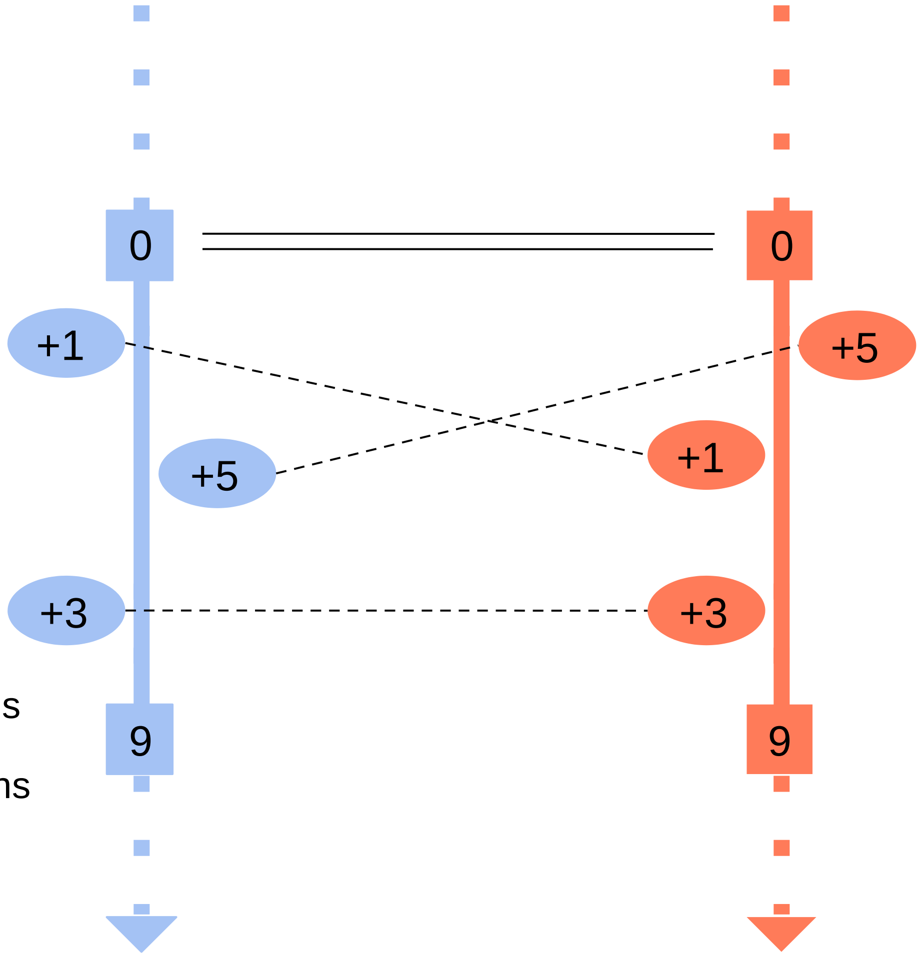

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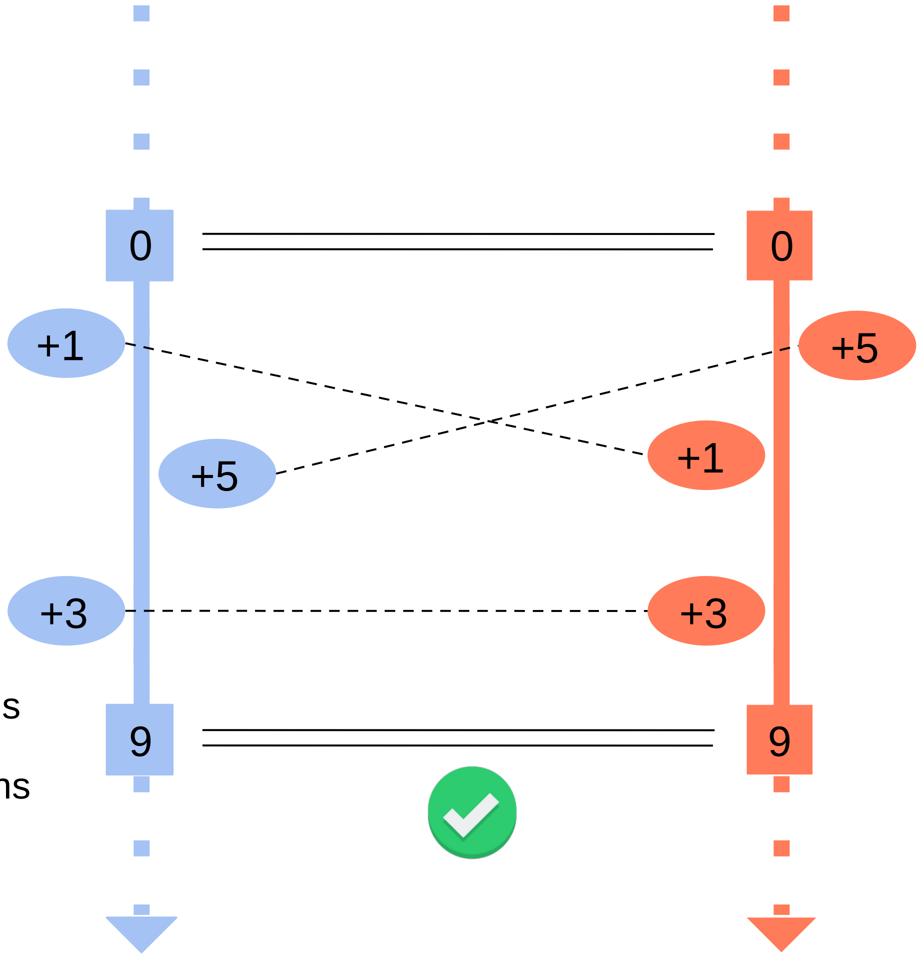

Basic Solution

shared = I

```
atomic:                 ||  atomic:
  shared = A            ||    shared = B
atomic:                 ||  if h > 0:
  shared = C            ||    atomic:
                        ||      shared = B
                        ...
```

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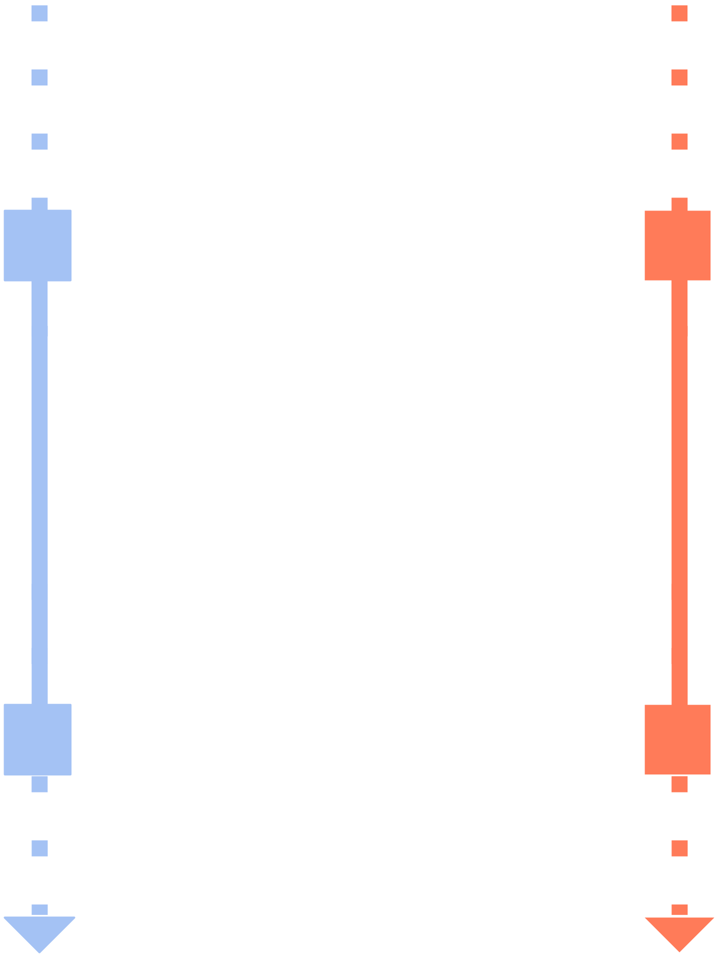

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              shared = I
atomic:                    ||  atomic:
   shared = A              ||     shared = B
atomic:                    ||  if h > 0:
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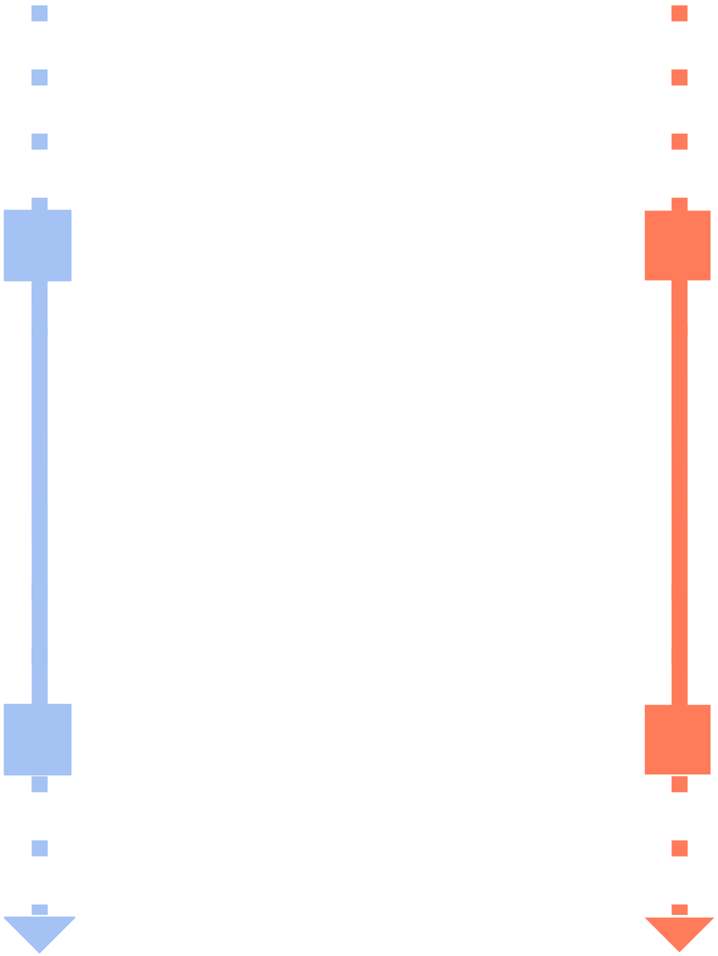

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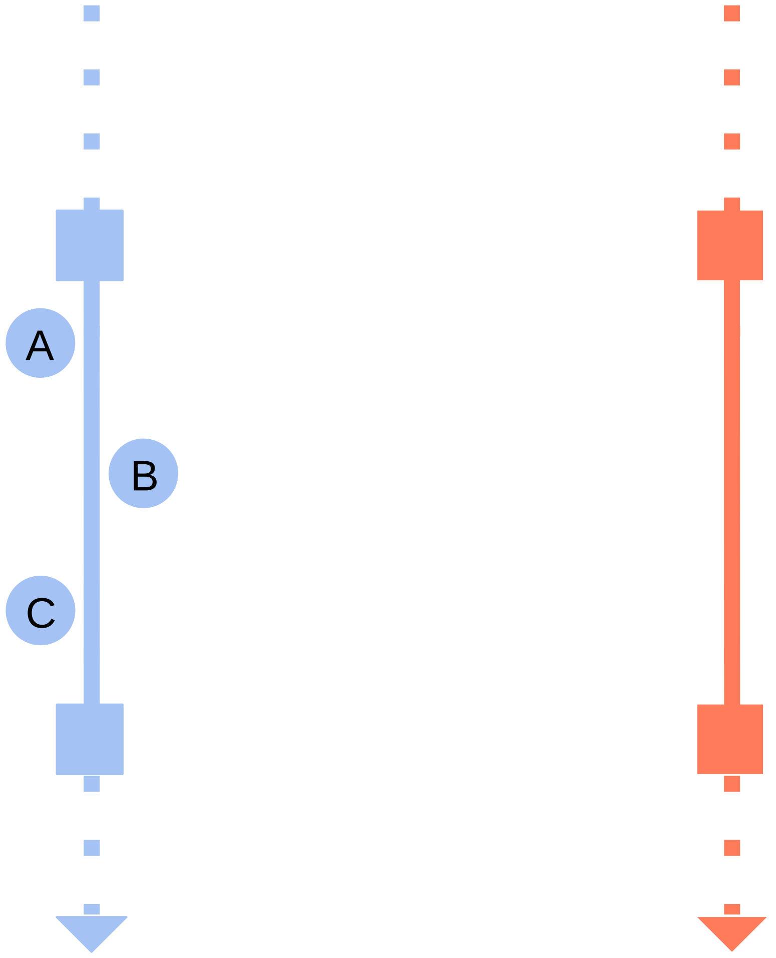

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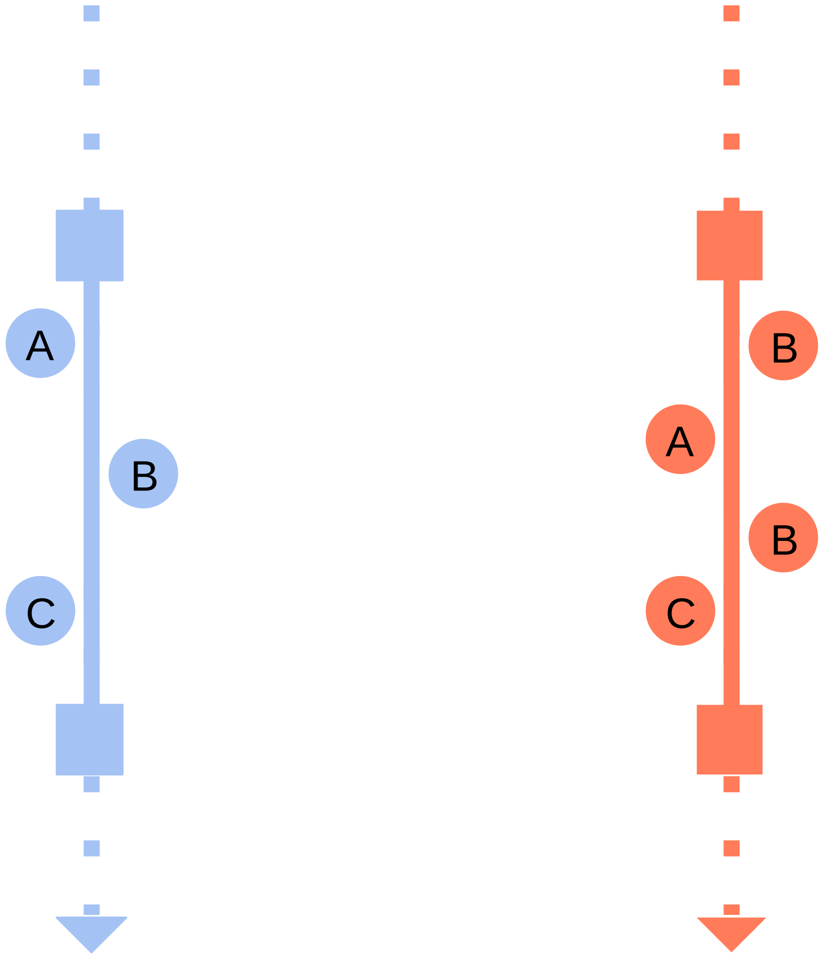

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            shared = I
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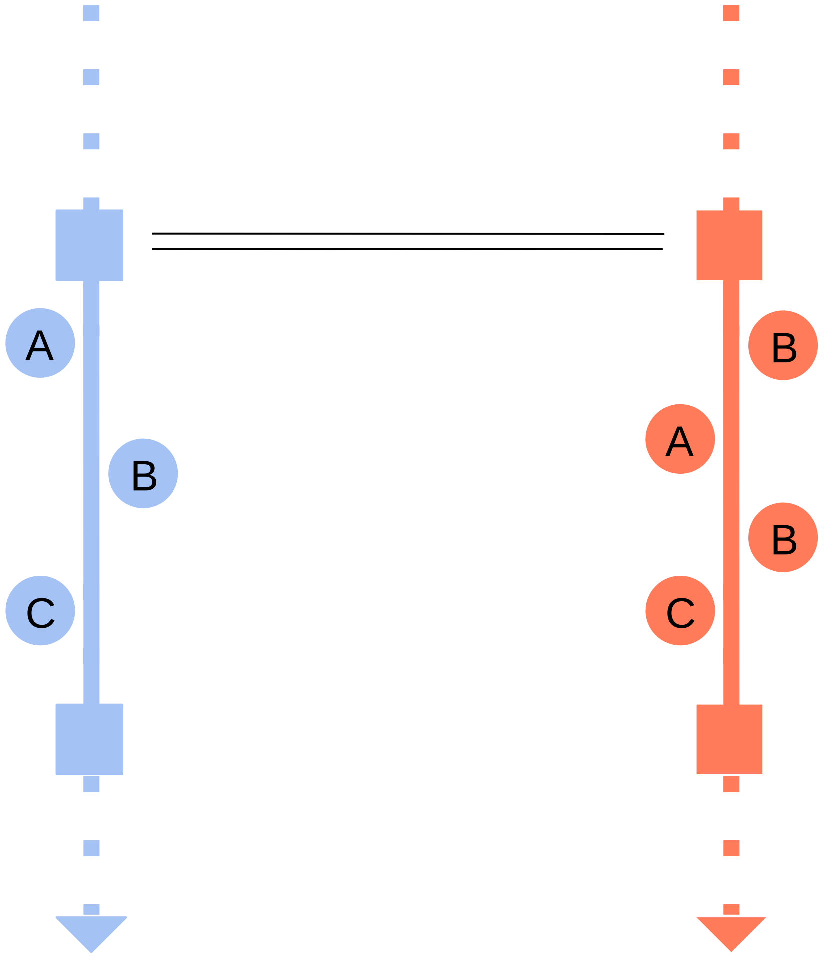

Basic Solution

shared = l

```
atomic:                  ||   atomic:
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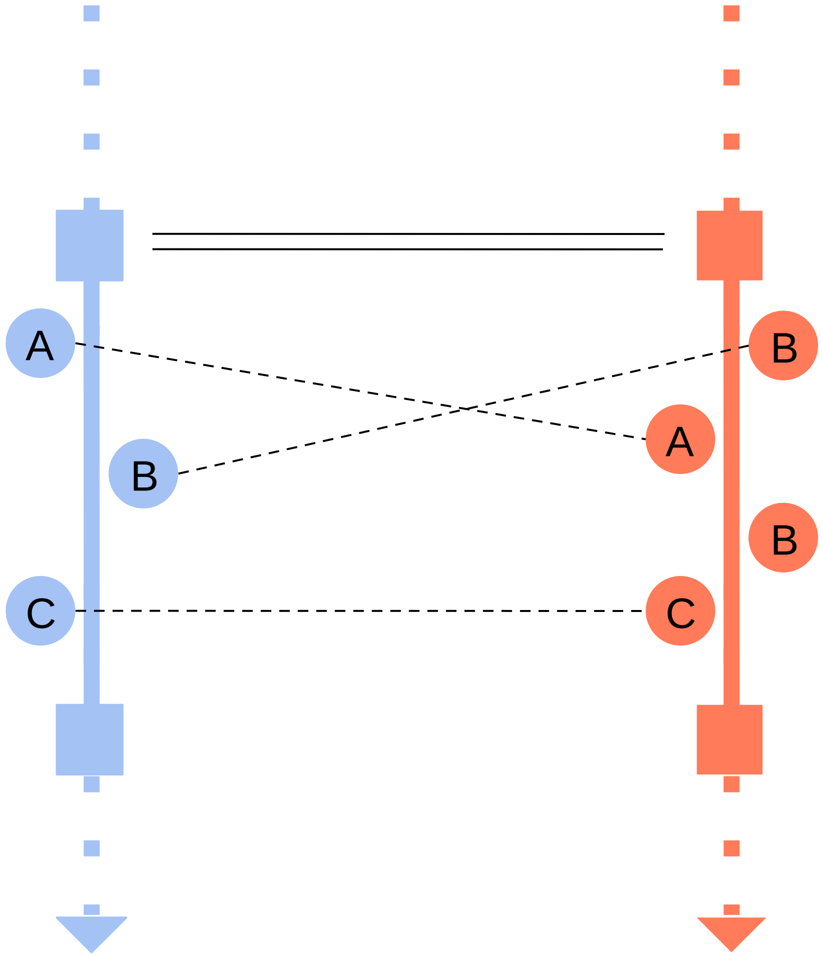

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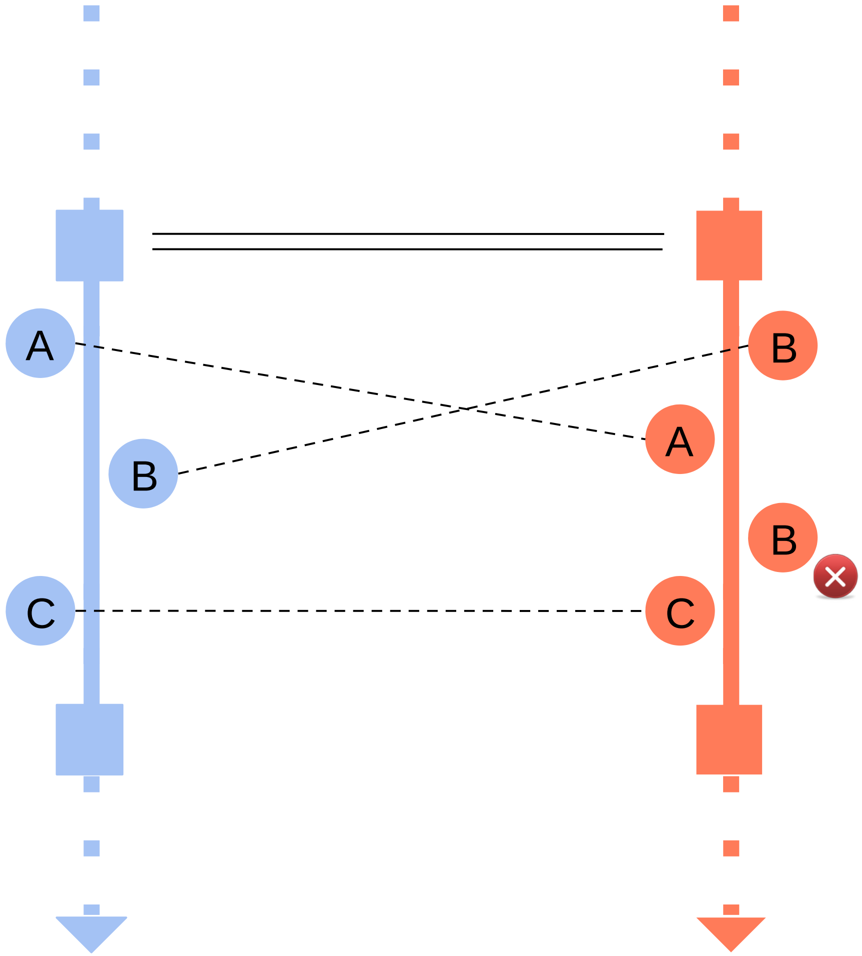
```

If

- *shared* has the same initial value in both executions
- (2) the two executions perform the “same” modifications
- (3) the modifications commute

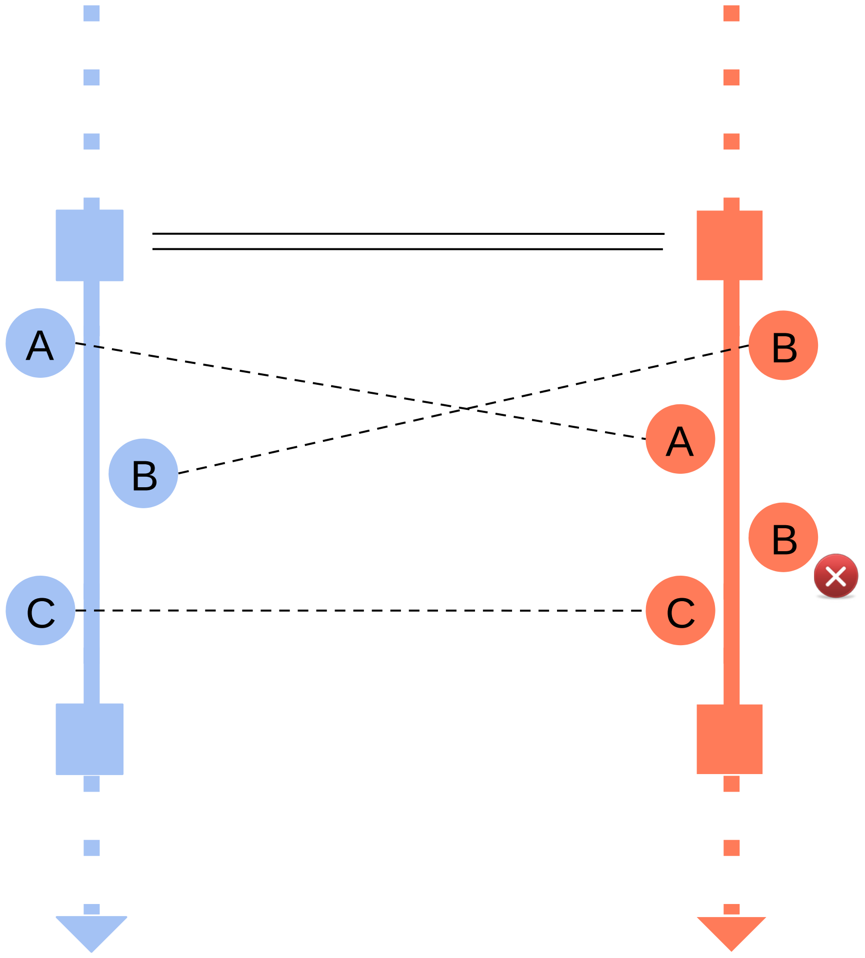
then *shared* has the same final value in both executions



Basic Solution

```
shared = I

atomic:              ||  atomic:
  shared = A         ||    shared = B
atomic:              ||  if h > 0:
  shared = C         ||    atomic:
                     ||      shared = B
                    ...
```



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                       ...
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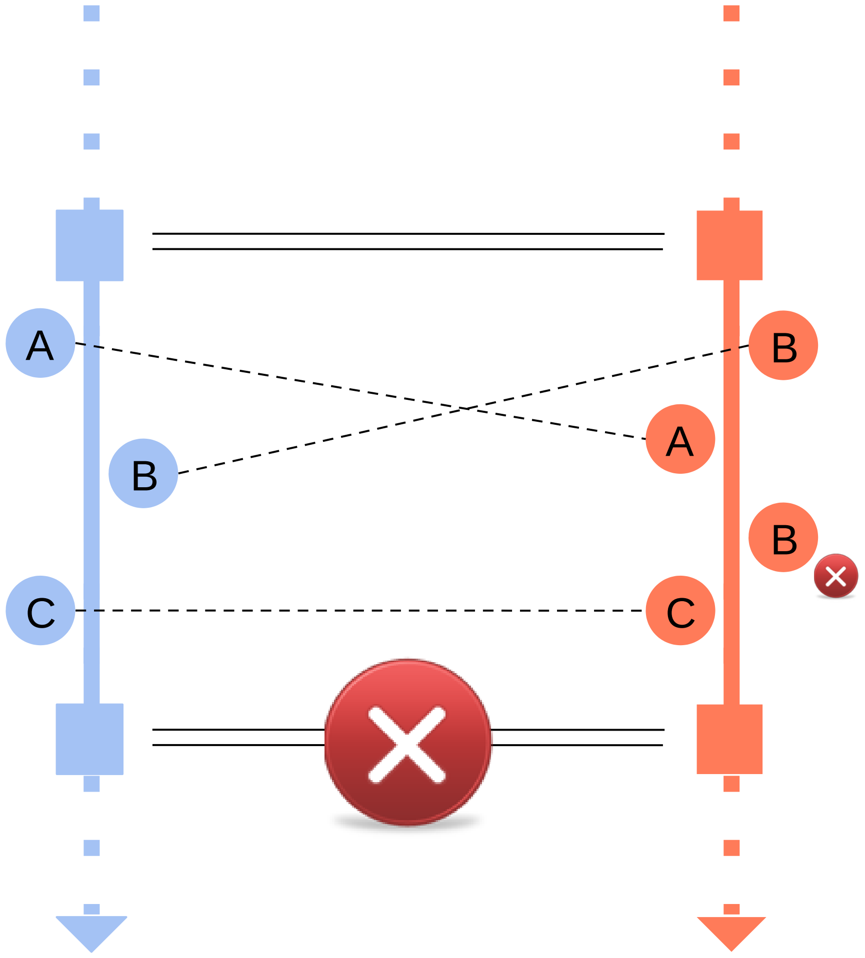
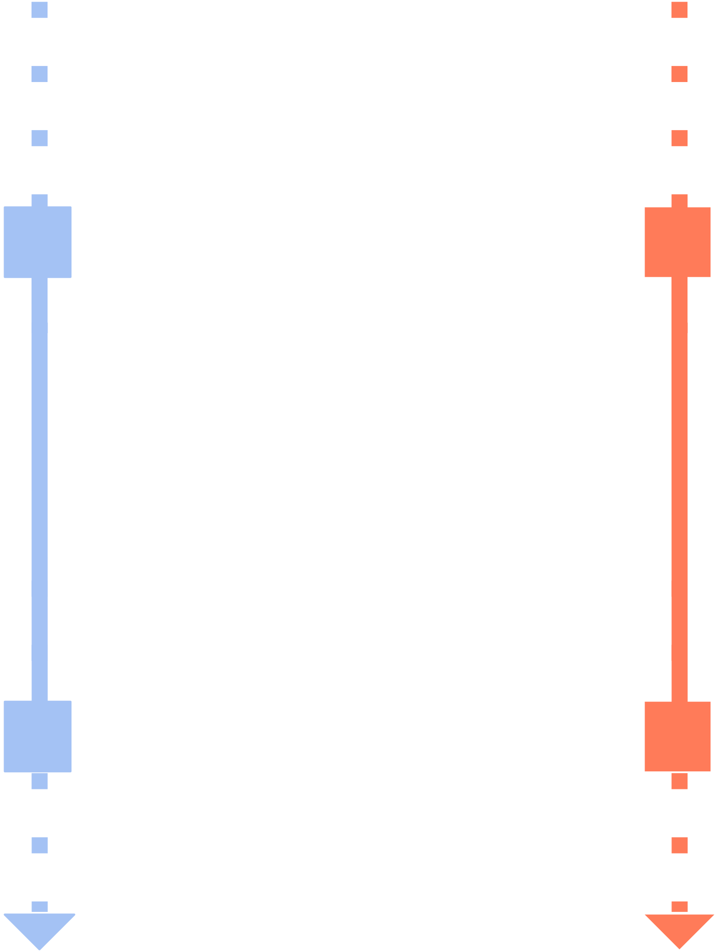


Basic Solution

shared = I

```
atomic:                    ||  atomic:
  shared *= ...            ||    shared += ...
                           ||
atomic:                    ||
  shared += ...            ||
             ...
```



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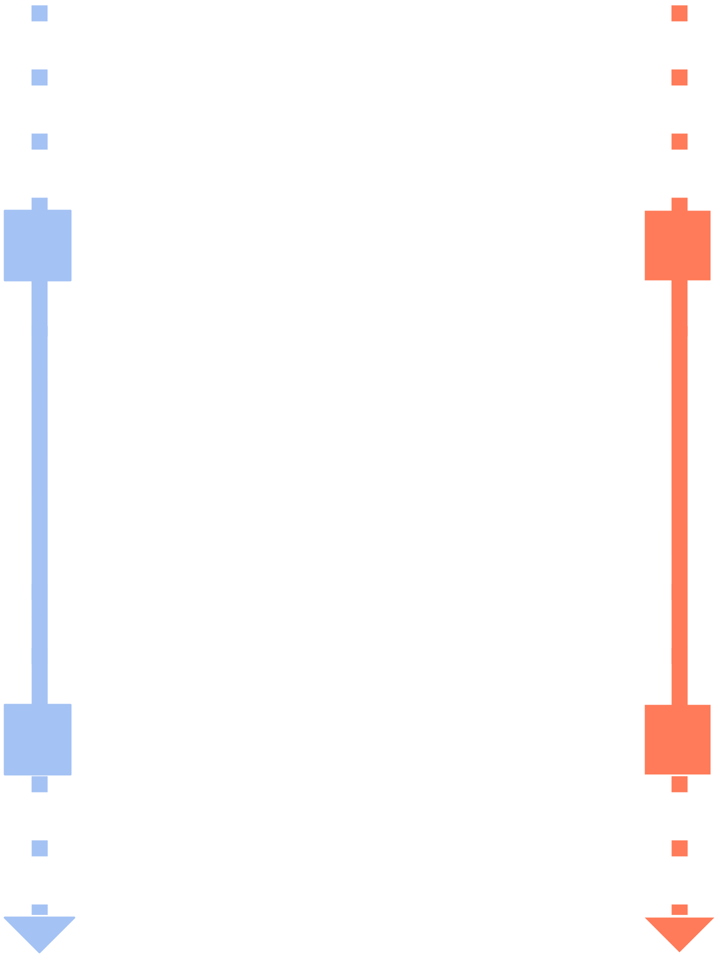

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                        ||
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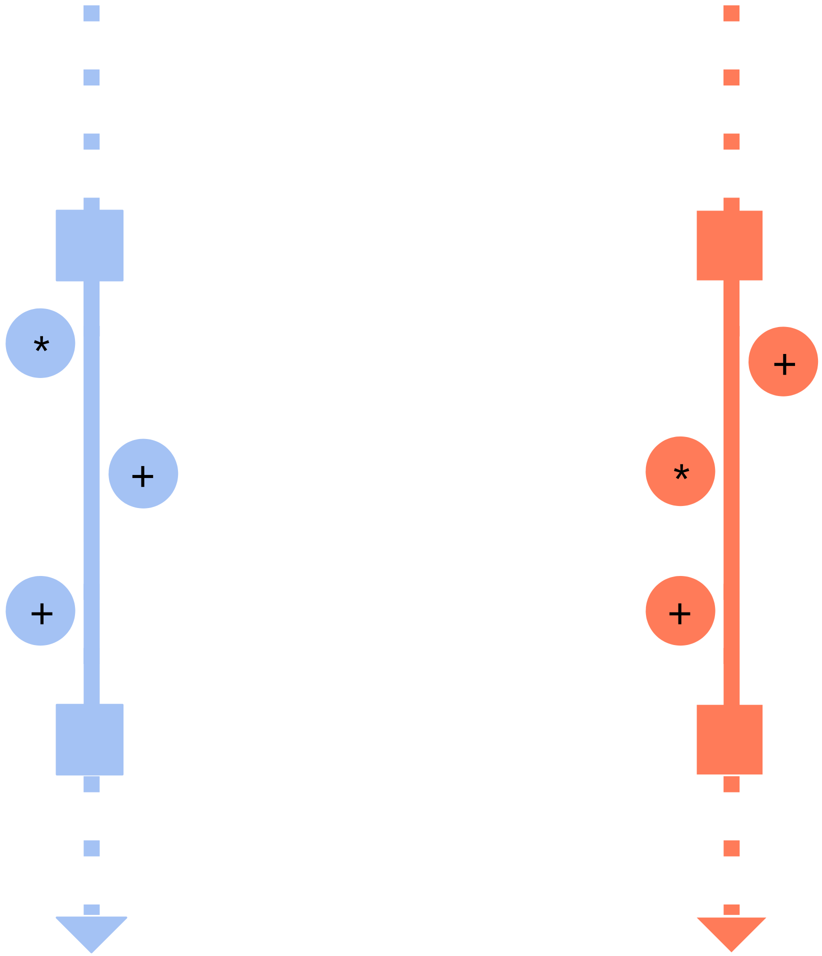

Basic Solution

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atomic:                   ||  atomic:
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atomic:                   ||
  shared += ...           ||
            ...
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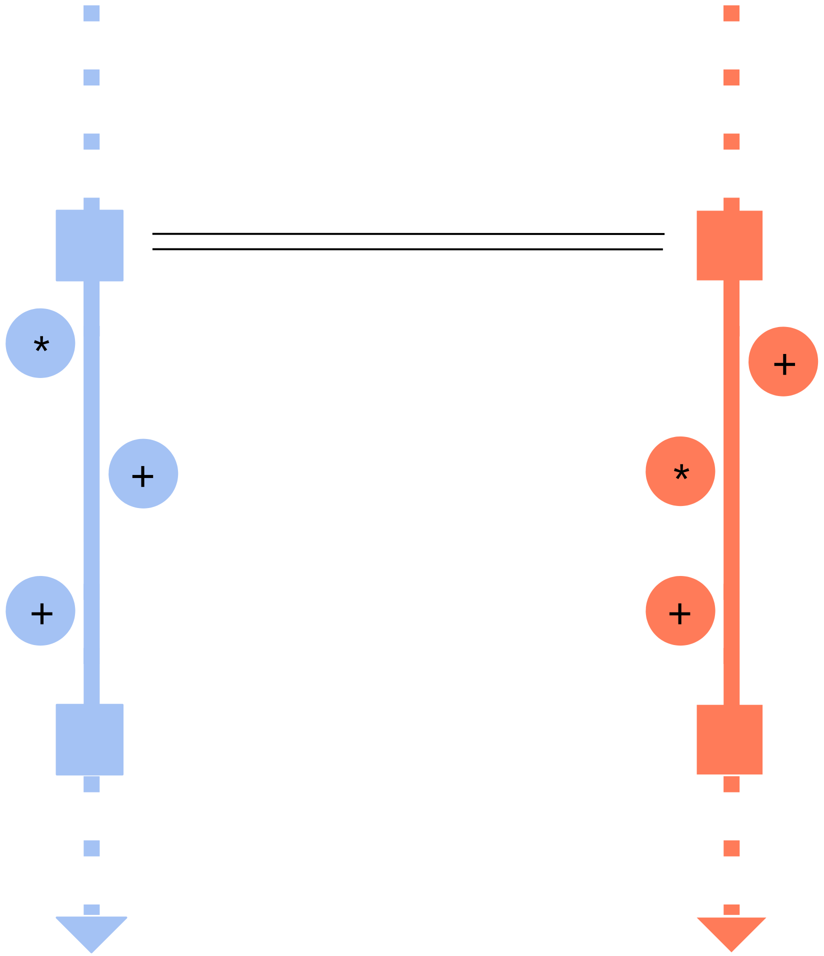

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atomic:
  shared += ...          ||
               ...
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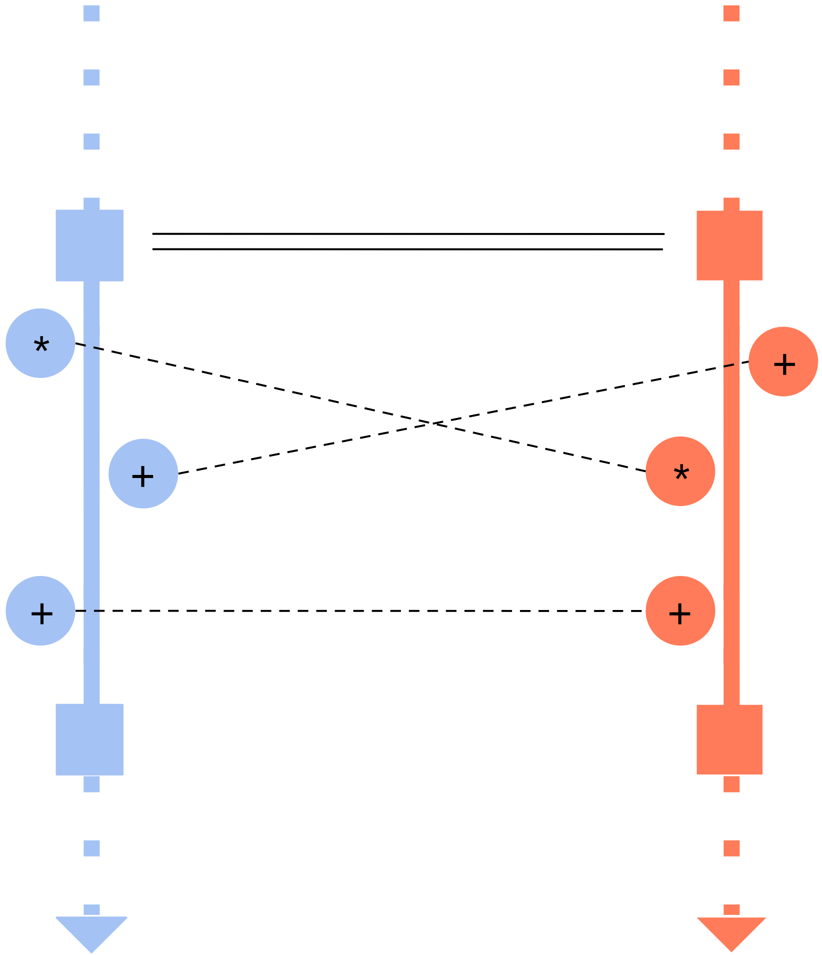

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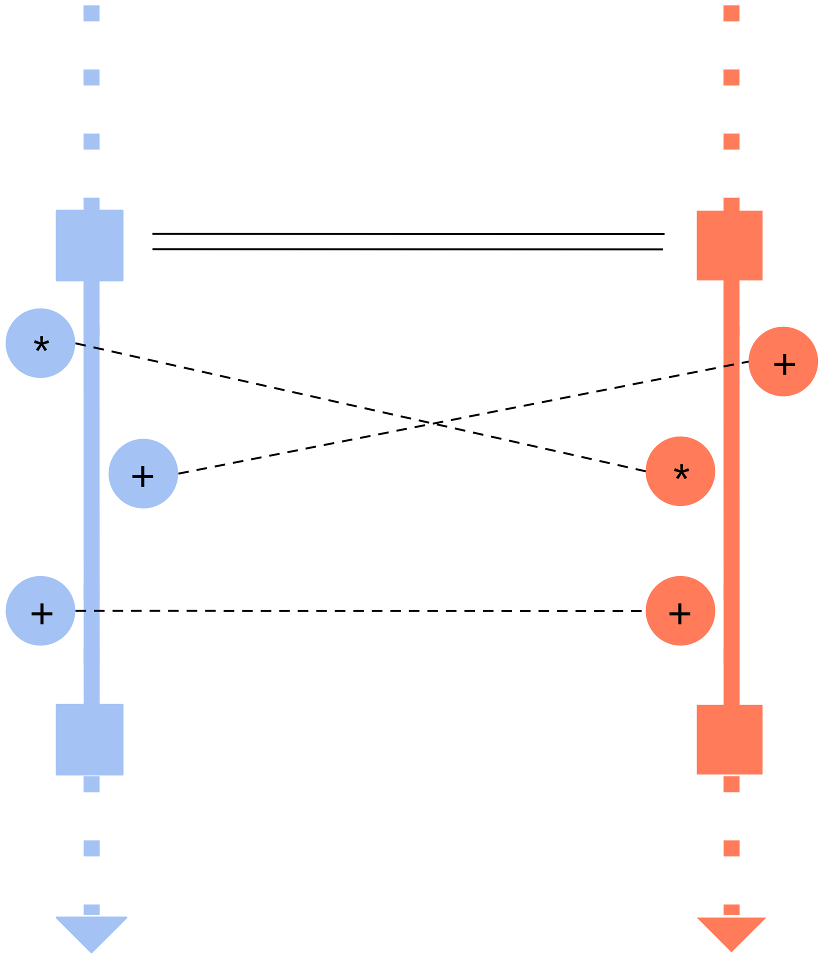

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                           ||
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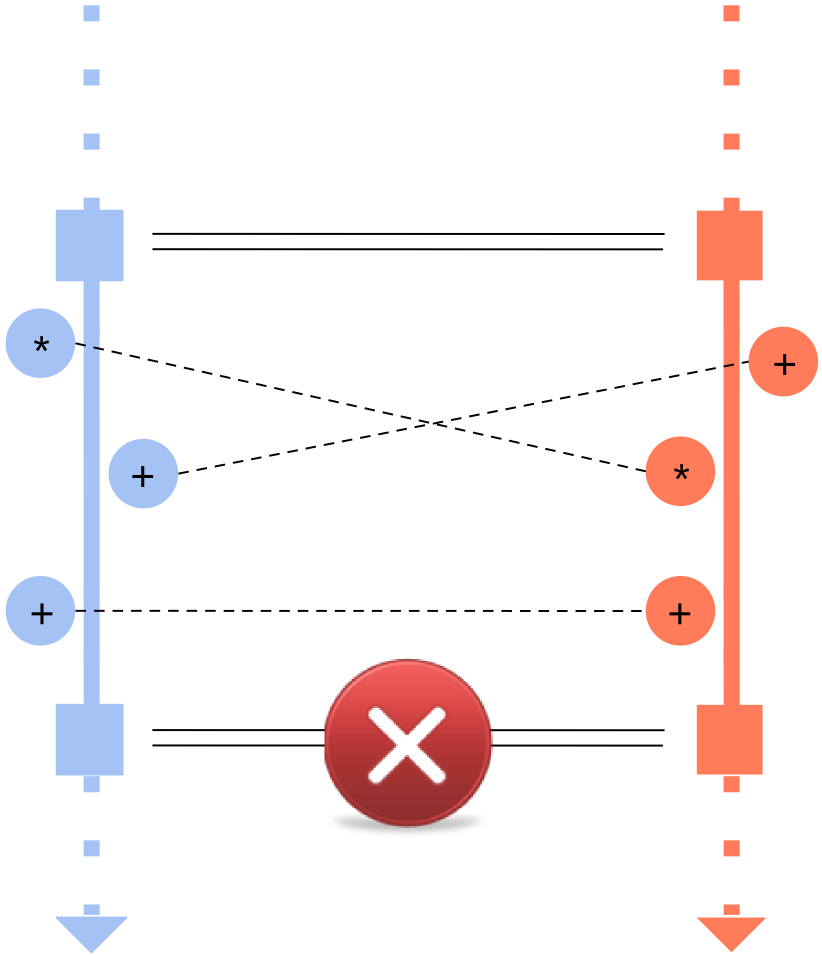

Back to Program Verification

Verification Approach

Verification Approach

Based on **Concurrent Separation Logic** (CSL)

Verification Approach

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- Extension of Hoare Logic to concurrent heap-manipulating programs

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- Uses the notion of **resource ownership** (e.g., read/write permission)
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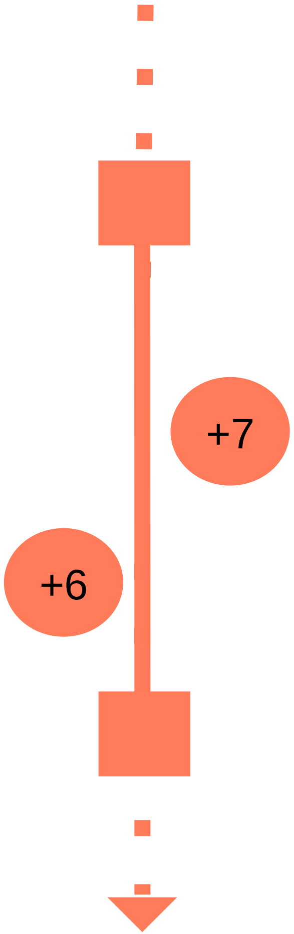
```
                shared = l
                  share
while i < h:           ||  while j < 100:
  i += 1               ||    j += 1

atomic:                ||  atomic:
  shared += 6          ||    shared += 7
                 unshare
              return shared
```



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                    ||
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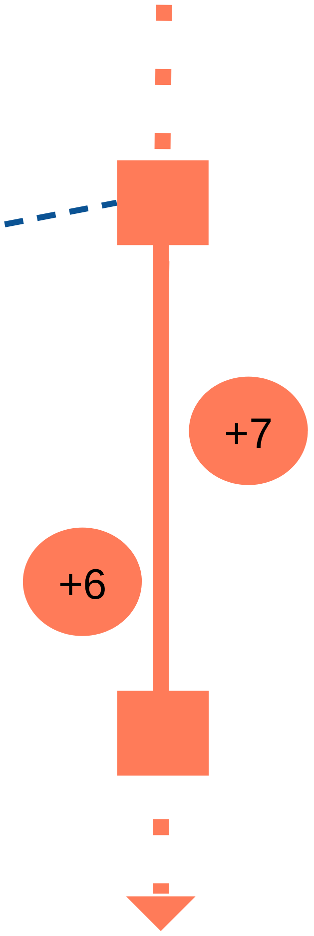
```



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              share
while i < h:          ||  while j < 100:
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                      ||
atomic:               ||  atomic:
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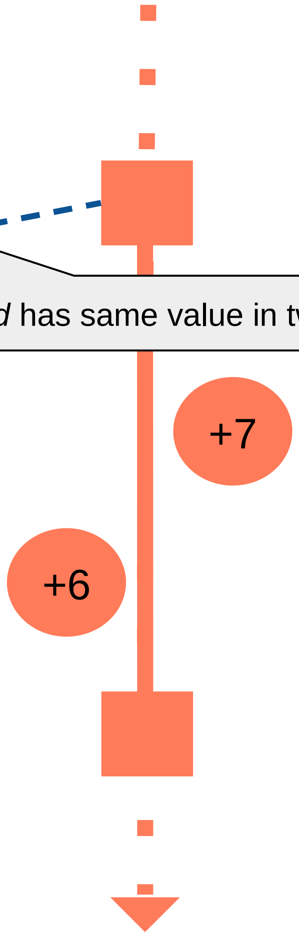
             unshare
           return shared
```

Prove 1) *shared* has same value in two executions



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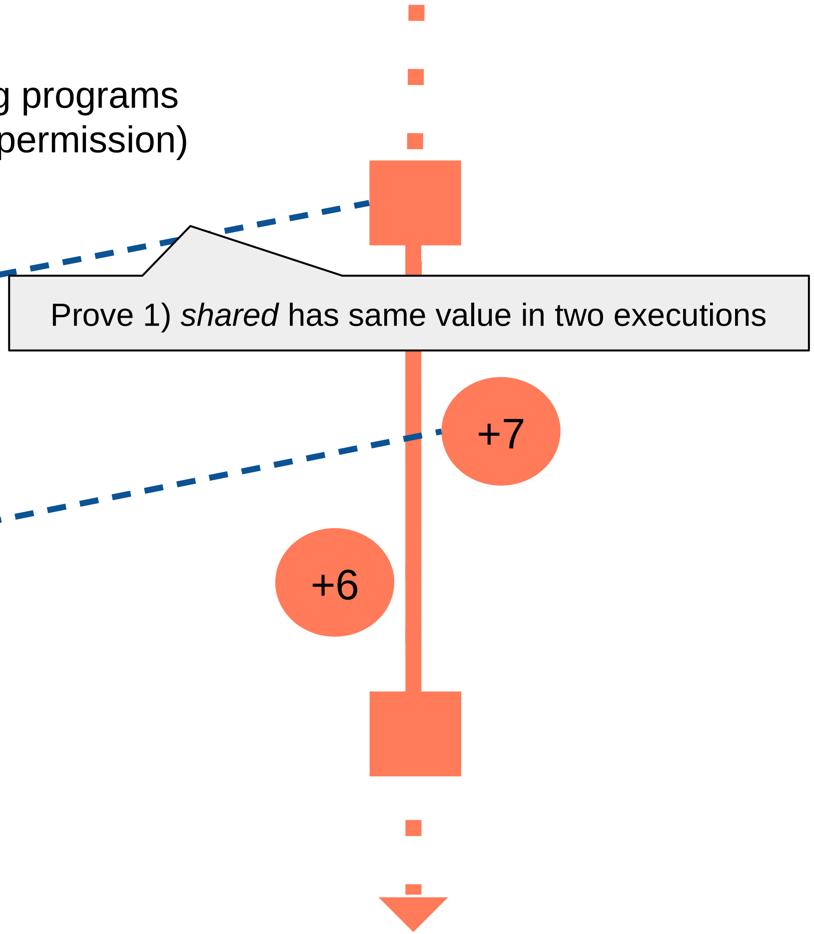
unshare
return shared
```



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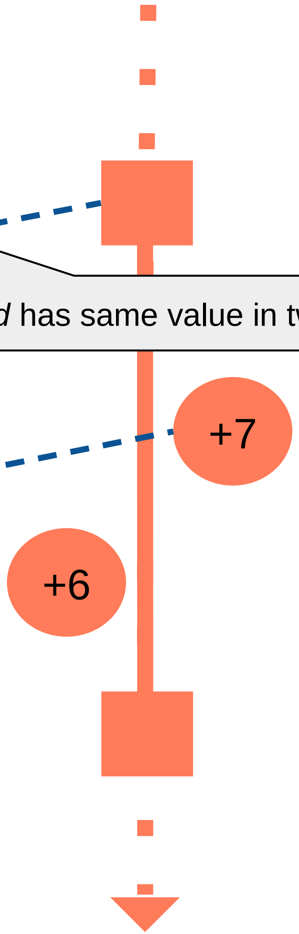
share

while i < h:            ||  while j < 100:
  i += 1                ||    j += 1
                        ||
atomic:                 ||  atomic:
  shared += 6           ||    shared += 7

unshare
return shared
```

Prove 1) *shared* has same value in two executions

Record each modification



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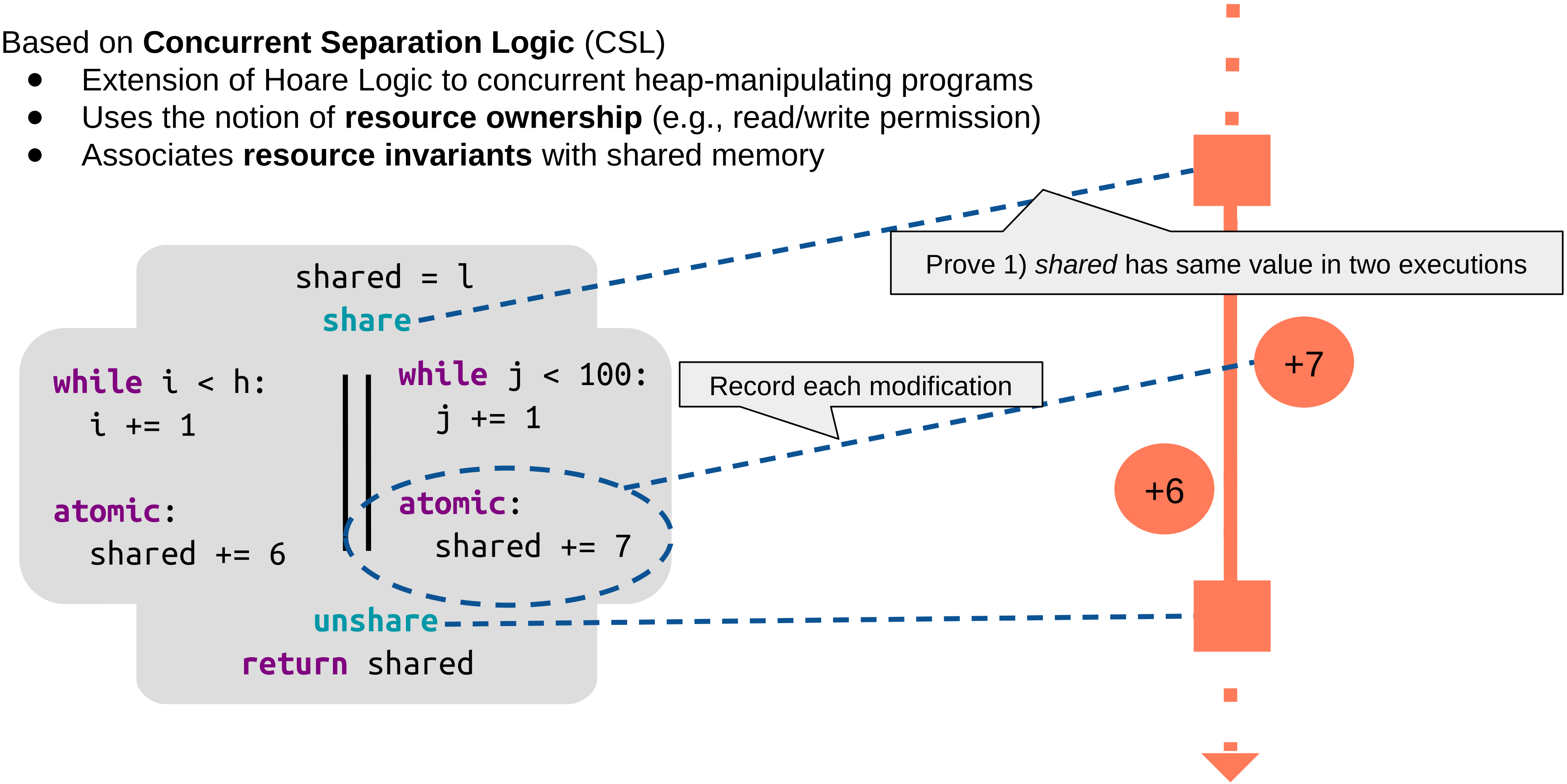

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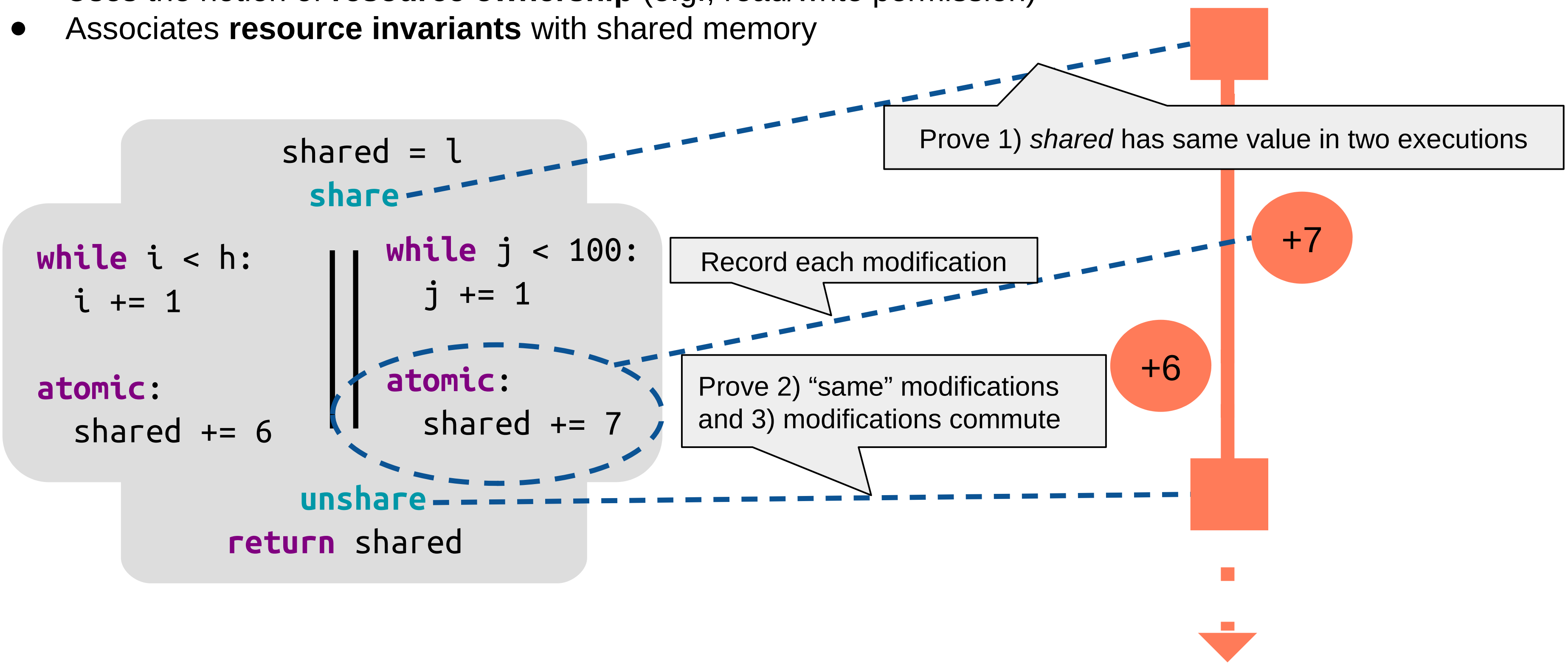

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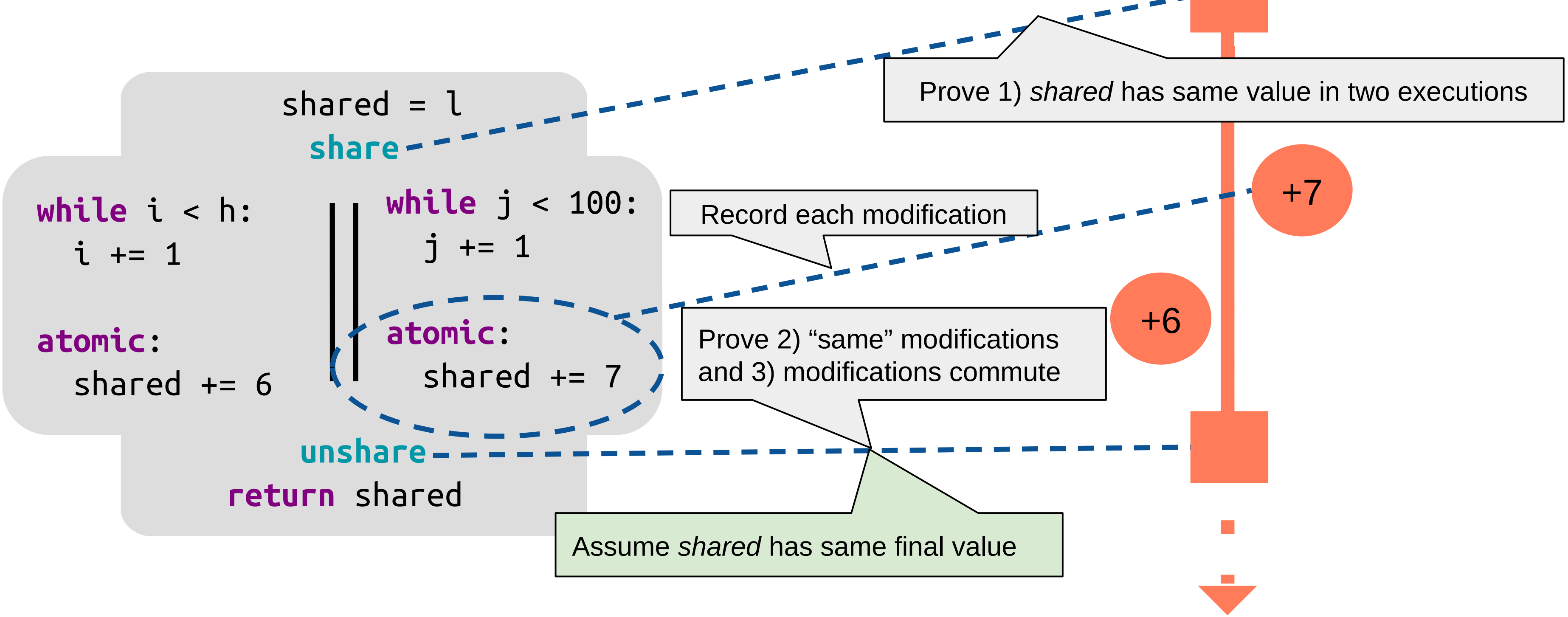


Verification Approach

```
shared = l

atomic:           ||  atomic:
  shared += 6     ||    shared += 7

return shared
```

Verification Approach

l has the same value in the two executions

```
{low(l)}
shared = l

atomic:              ||   atomic:
  shared += 6        ||     shared += 7

return shared
```

Verification Approach

l has the same value in the two executions

```
{low(l)}
shared = l

atomic:              ||   atomic:
  shared += 6        ||     shared += 7

return shared
{low(result)}
```

Verification Approach

l has the same value in the two executions

shared has the same value in the two executions (1)

```
{low(l)}
shared = l
{low(shared)}

atomic:            ||  atomic:
  shared += 6      ||    shared += 7

return shared
{low(result)}
```

Verification Approach

l has the same value in the two executions

shared has the same value in the two executions (1)

We use resources to record each modification

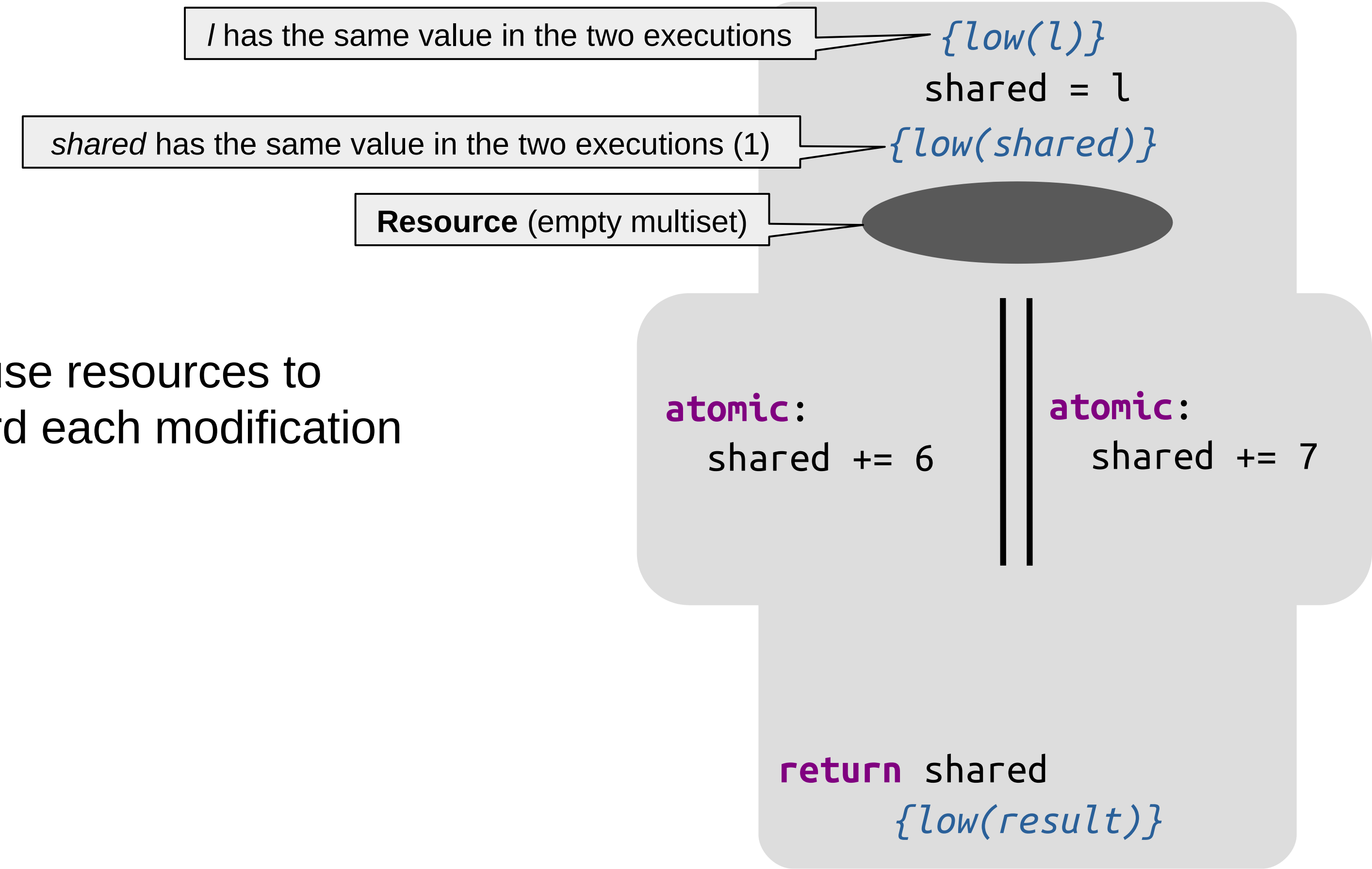
```
{low(l)}
shared = l
{low(shared)}

atomic:              ||  atomic:
  shared += 6        ||    shared += 7

return shared
{low(result)}
```

Verification Approach

We use resources to record each modification



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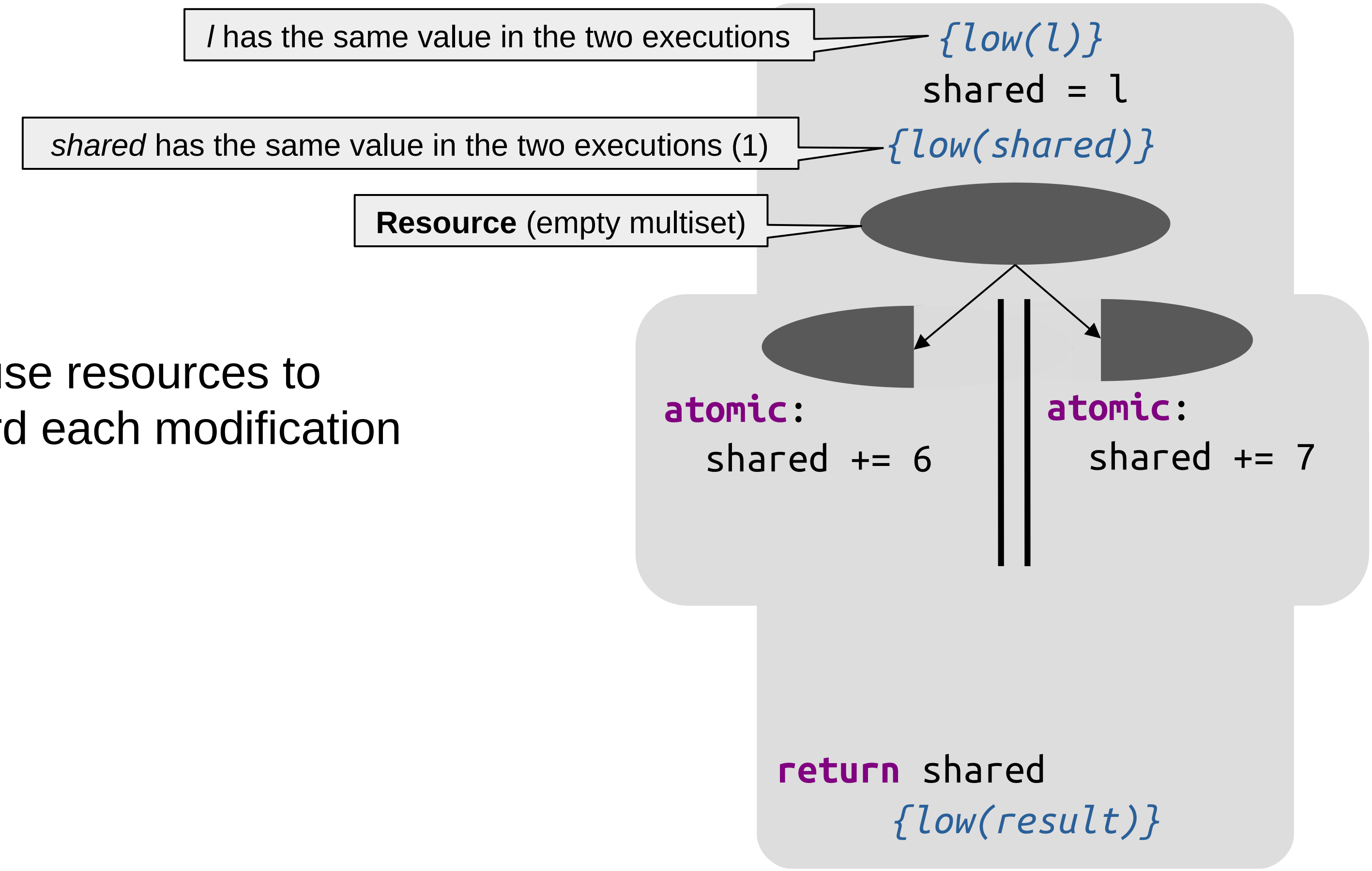

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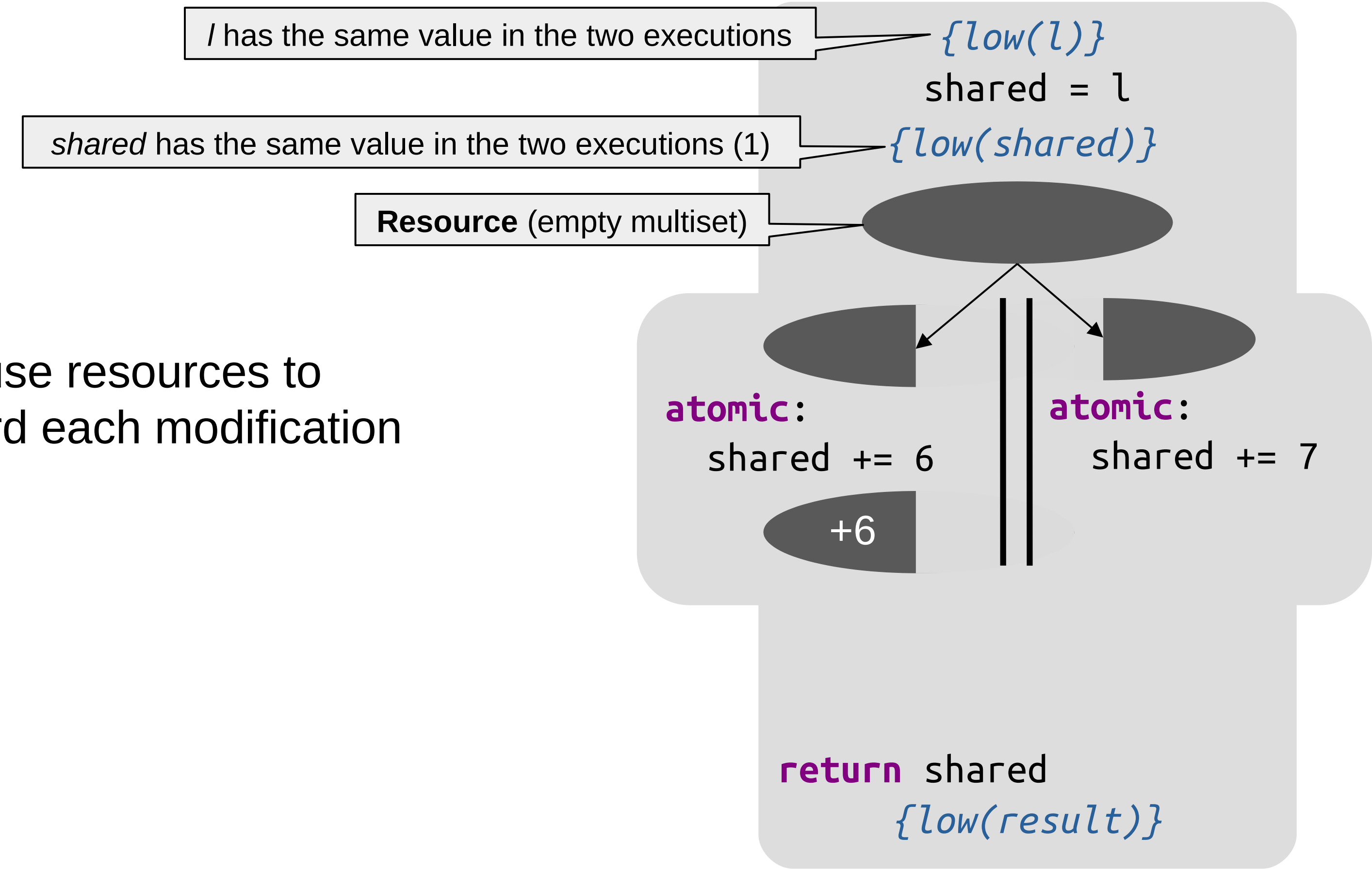

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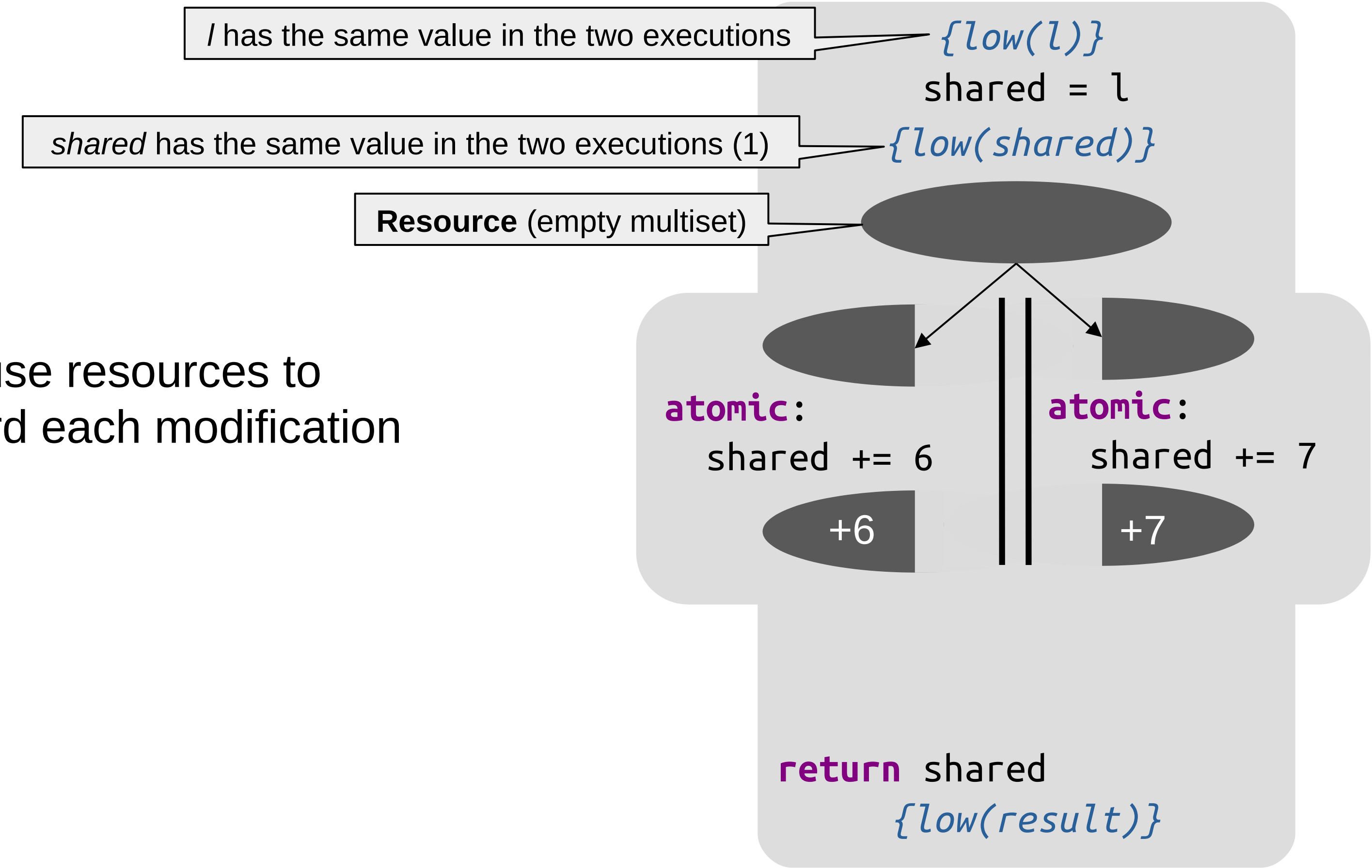

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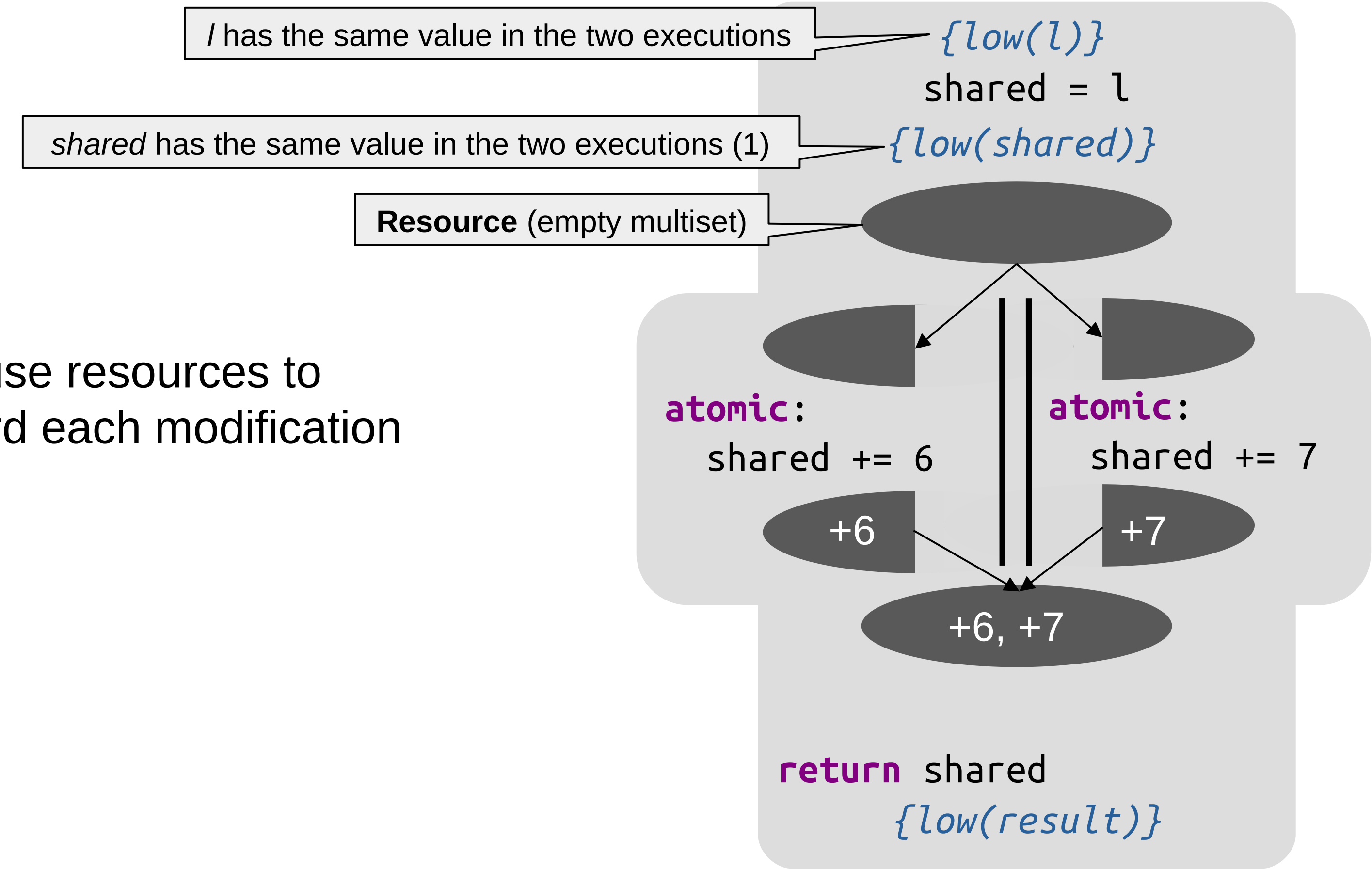

Verification Approach

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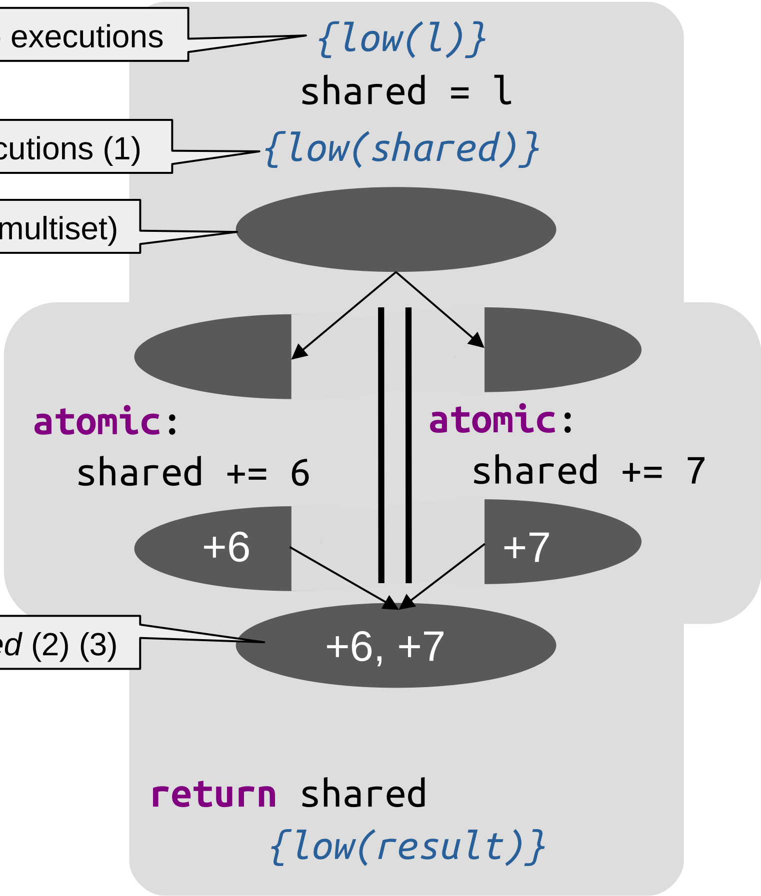

Verification Approach

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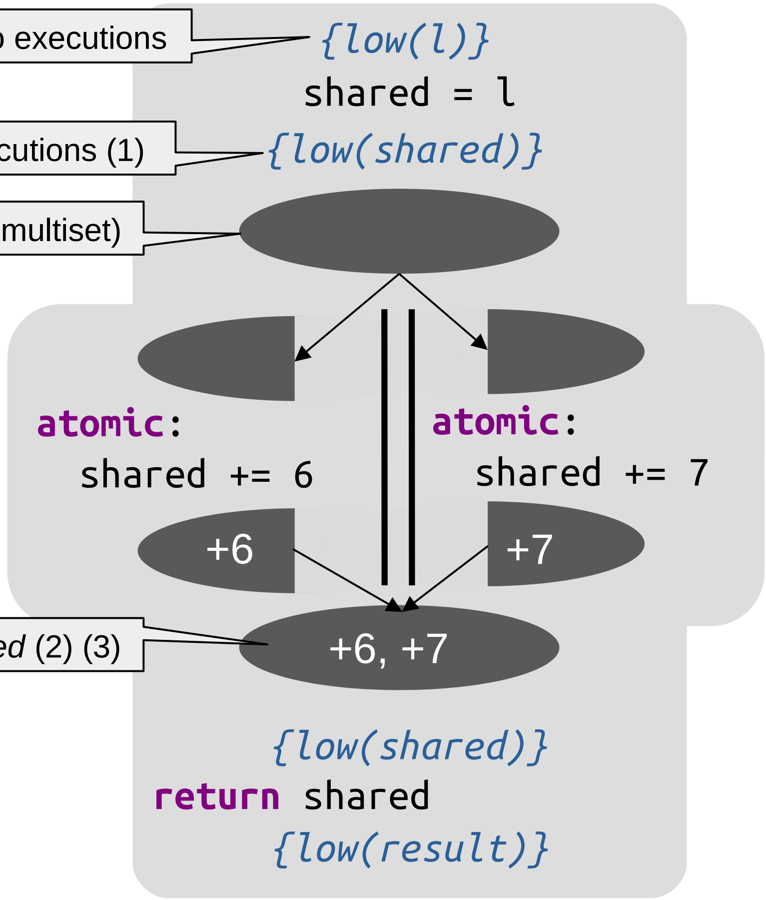
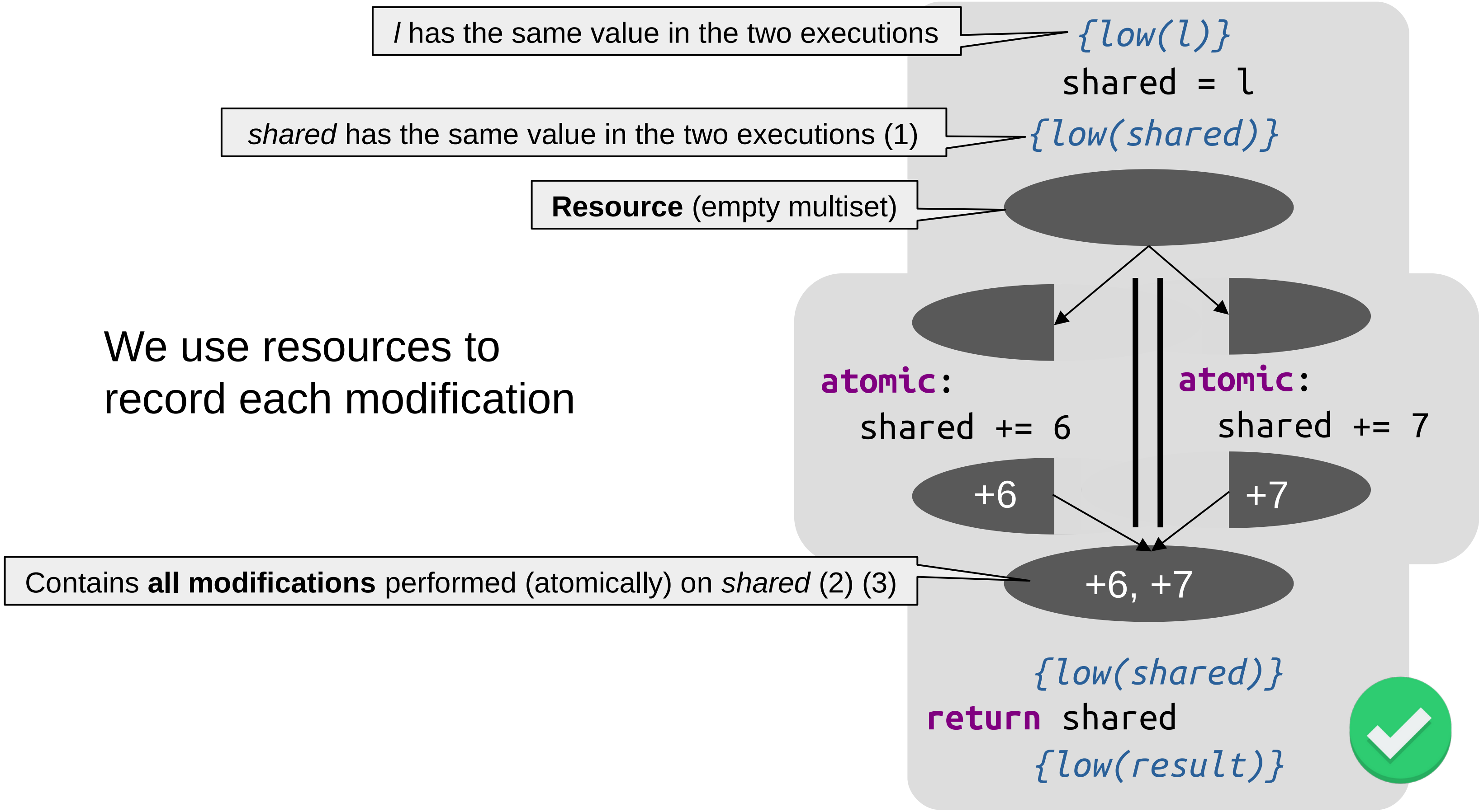


Verification Approach

We use resources to record each modification



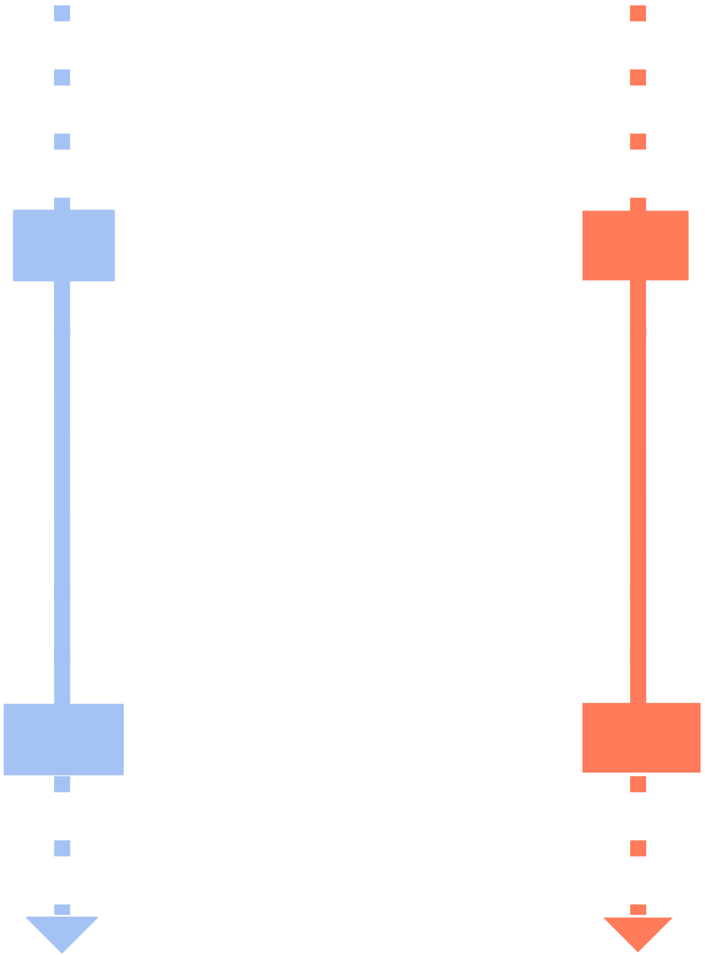
We can do better.

The Limits of Commutativity

```
shared = new List()

while i < h:          ||   while j < 100:
  i += 1              ||     j += 1
atomic:               ||   atomic:
  shared.add(6)       ||     shared.add(7)

return sort(shared)
```



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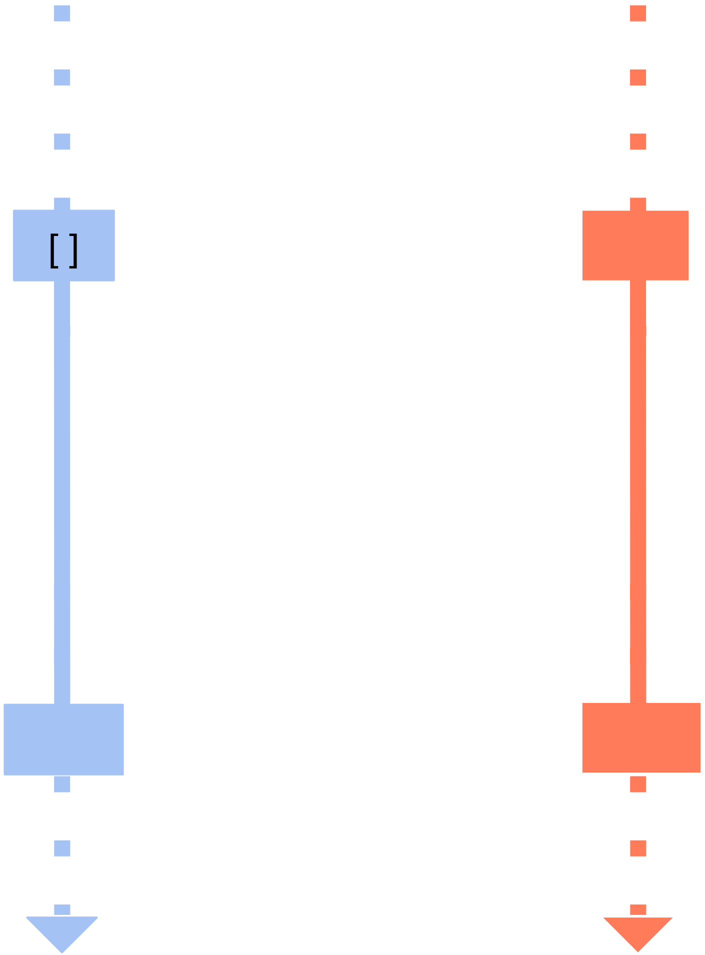

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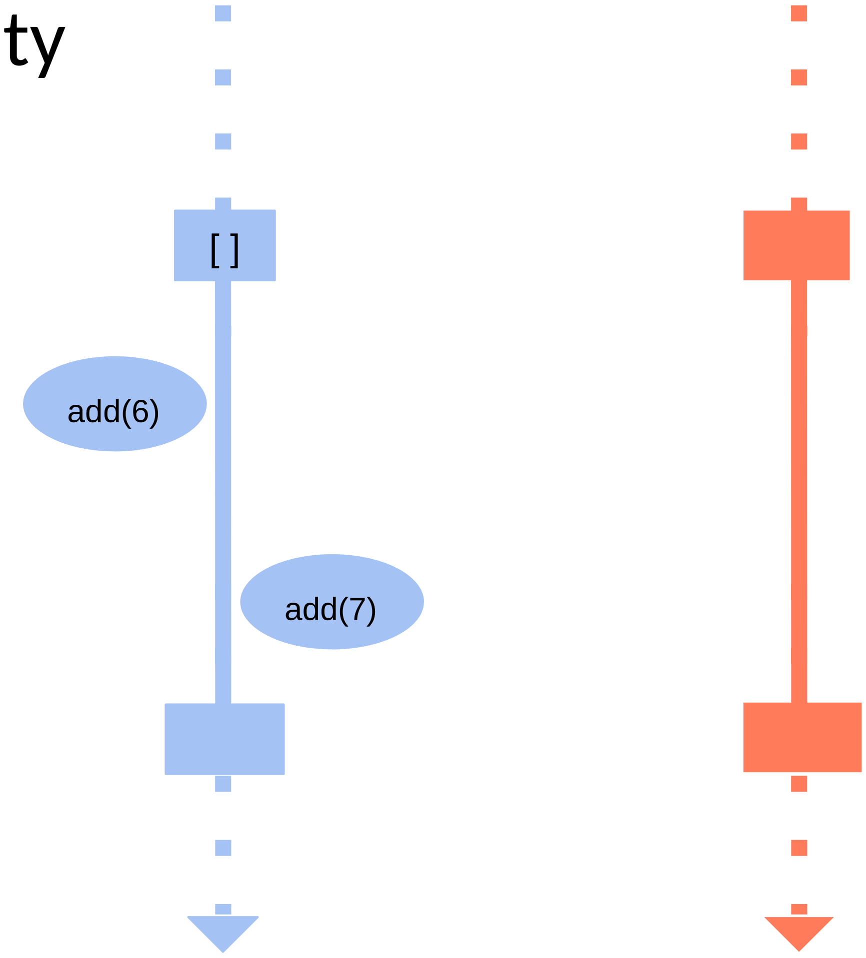

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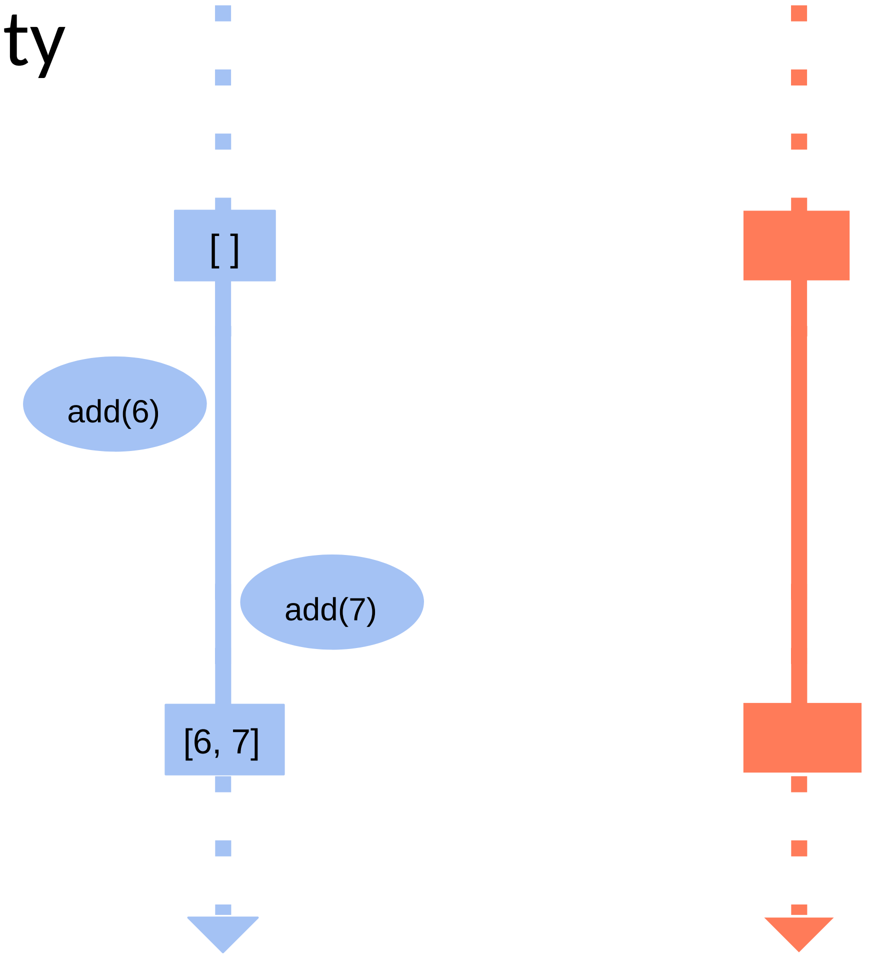

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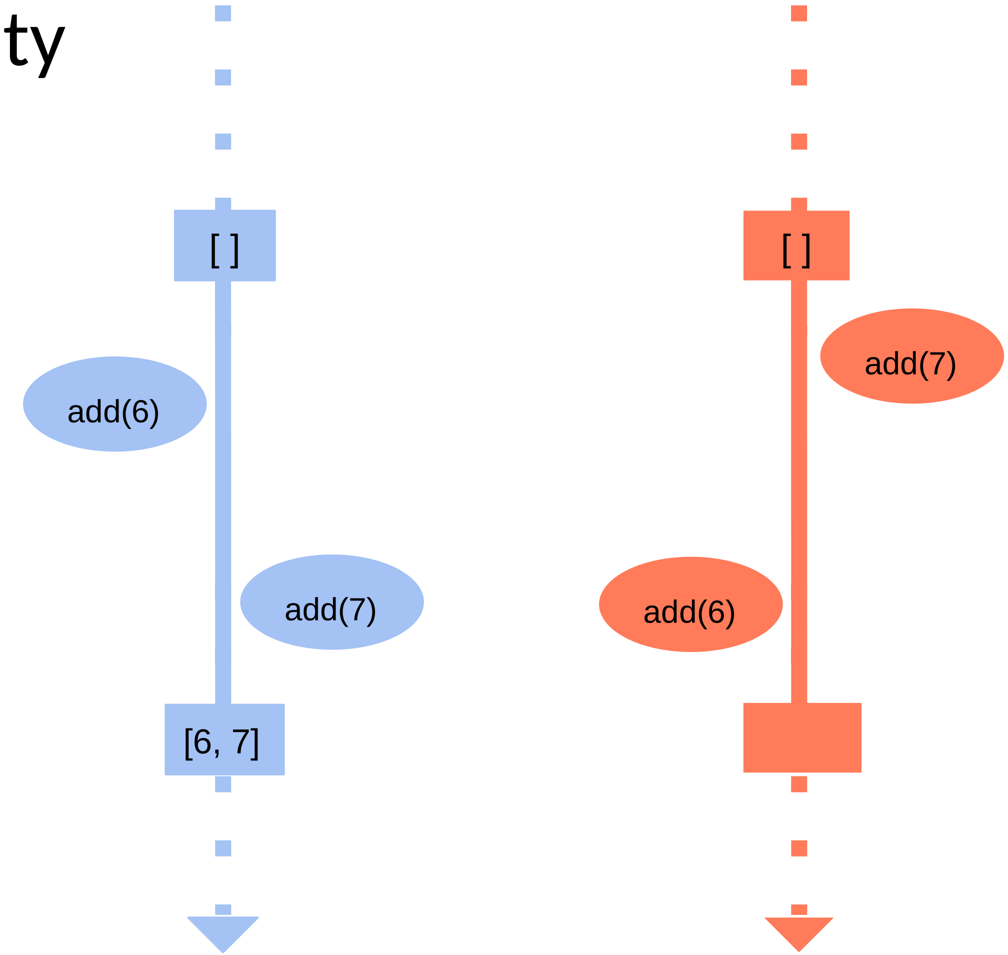

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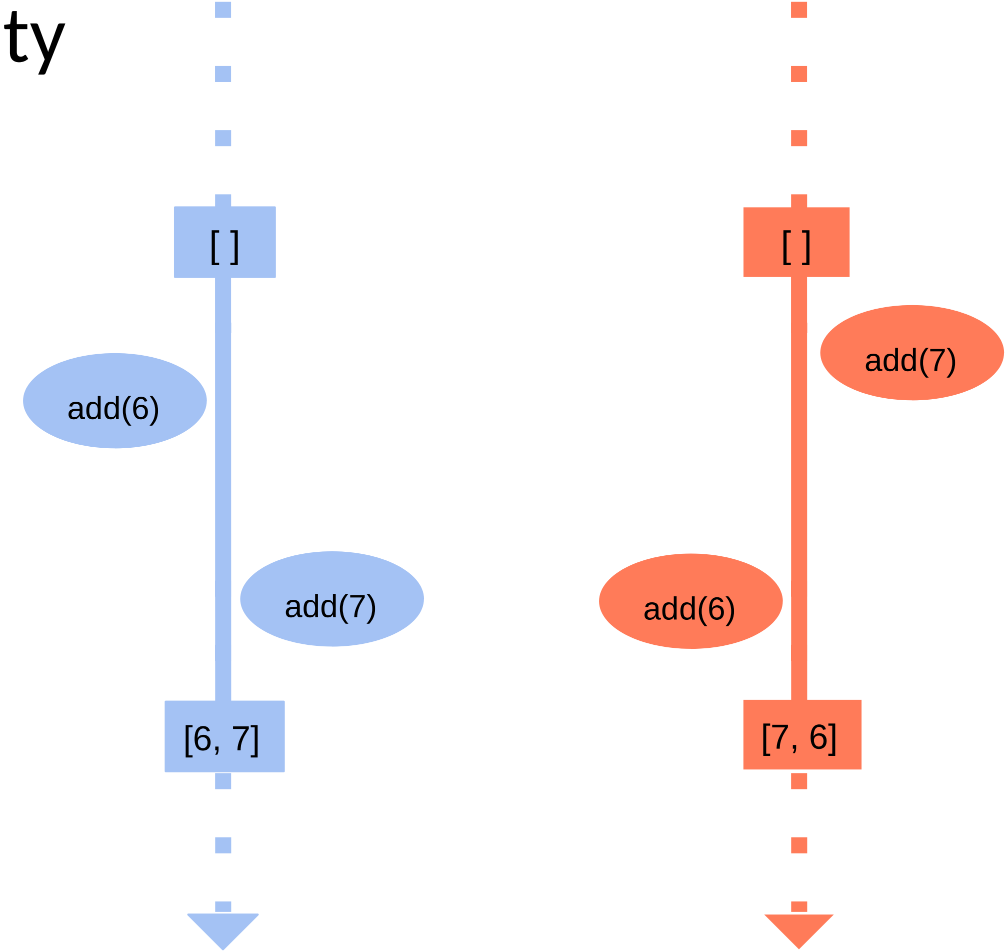

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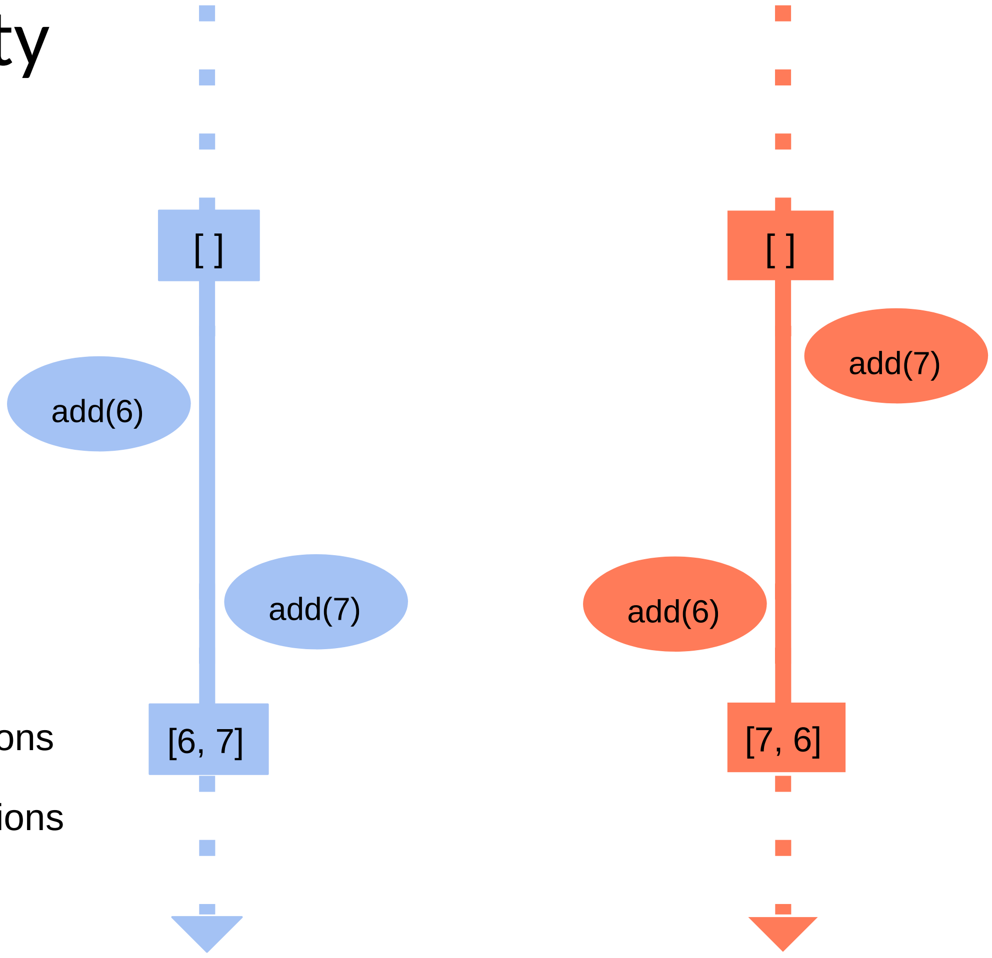

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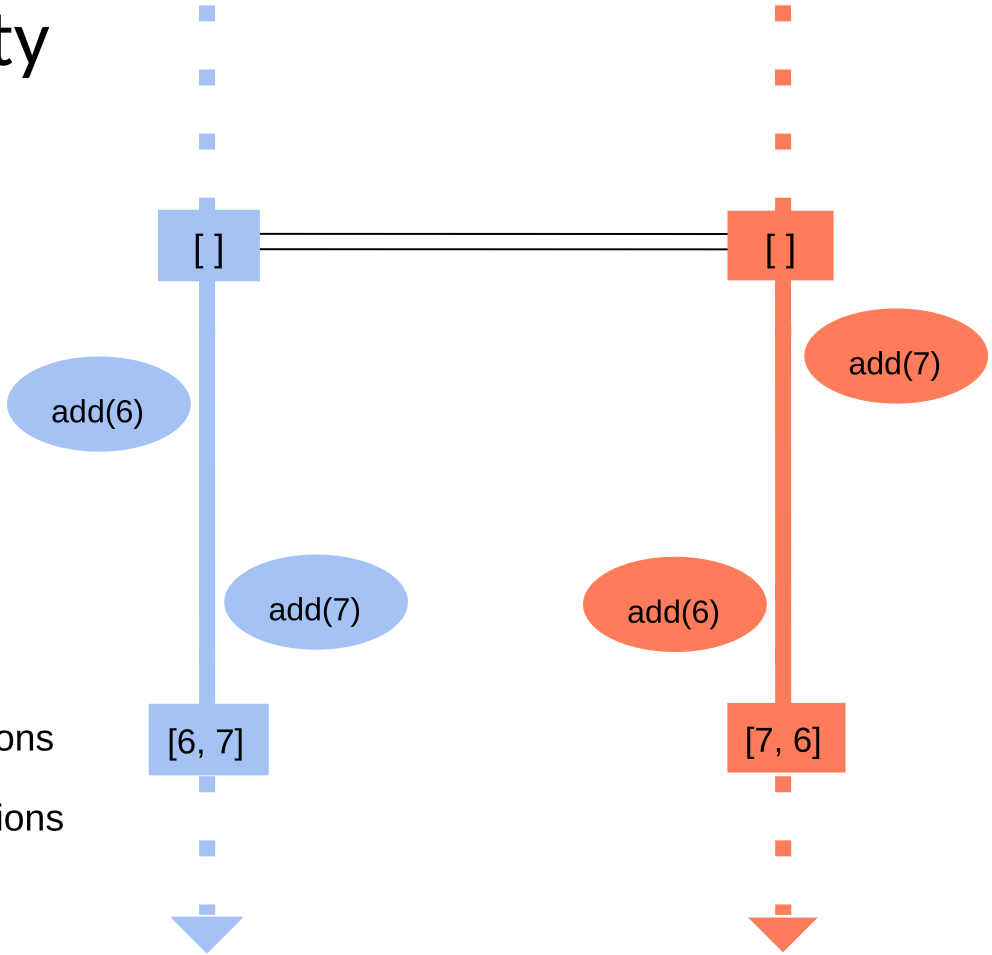

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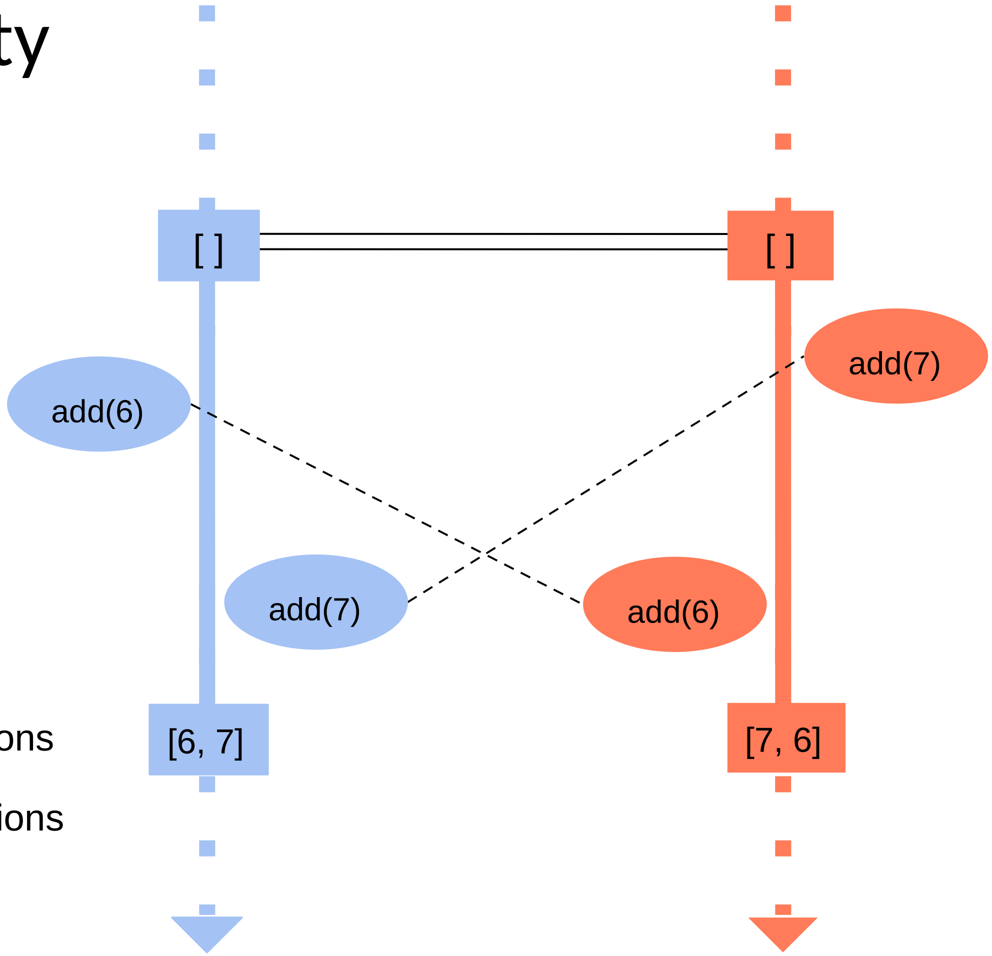

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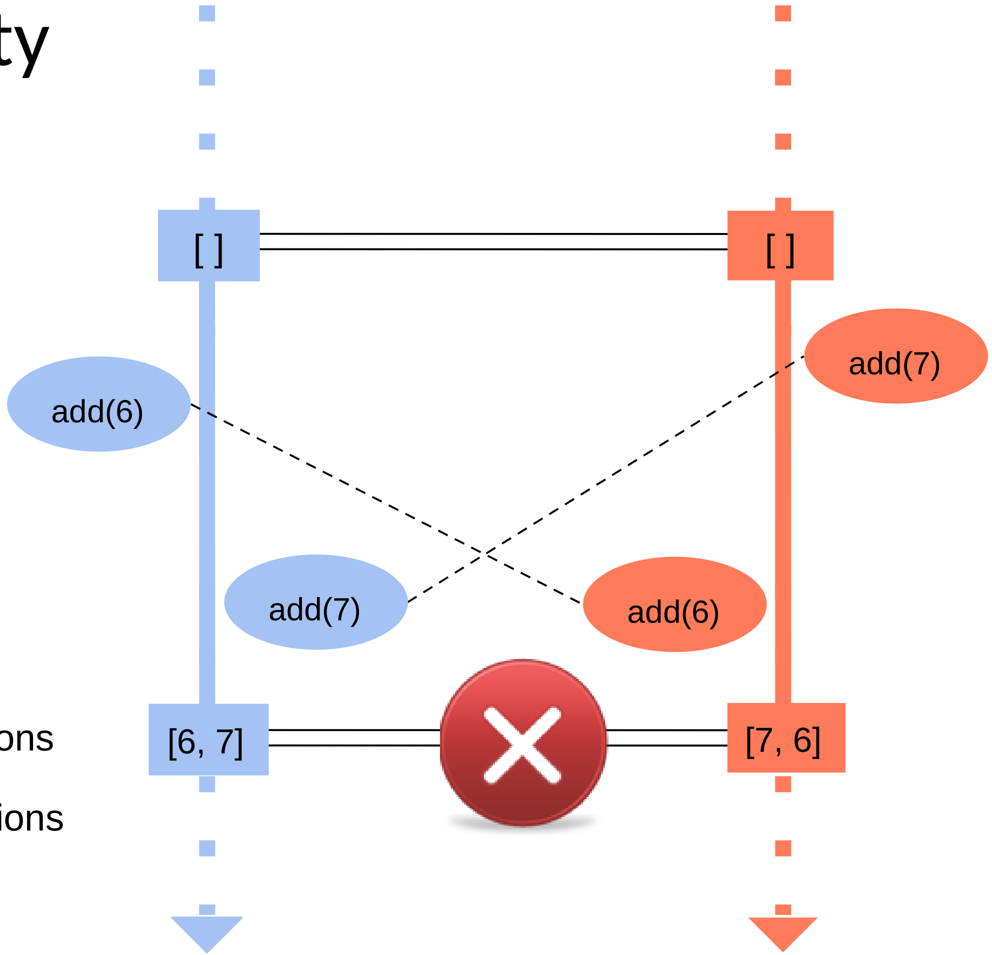

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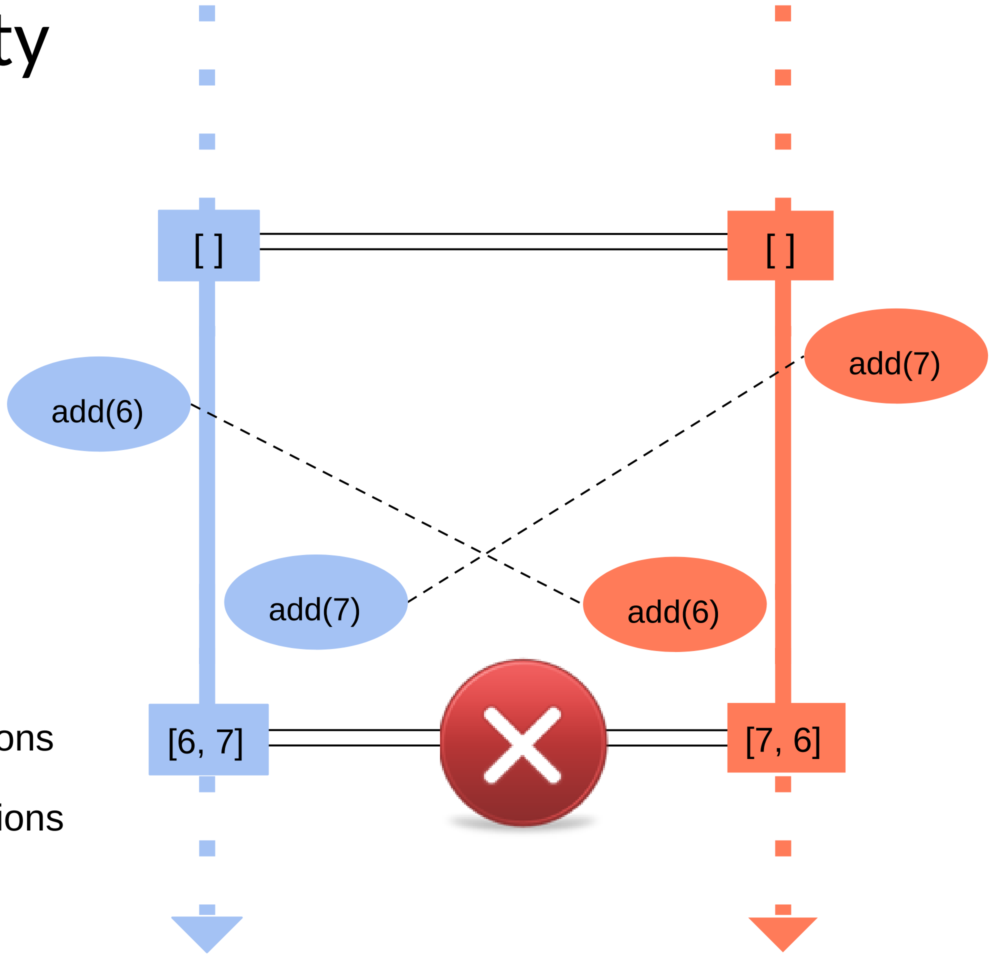
Secure



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Key Idea

Commutativity **modulo abstraction**

Commutativity Modulo Abstraction ("Abstract Commutativity")

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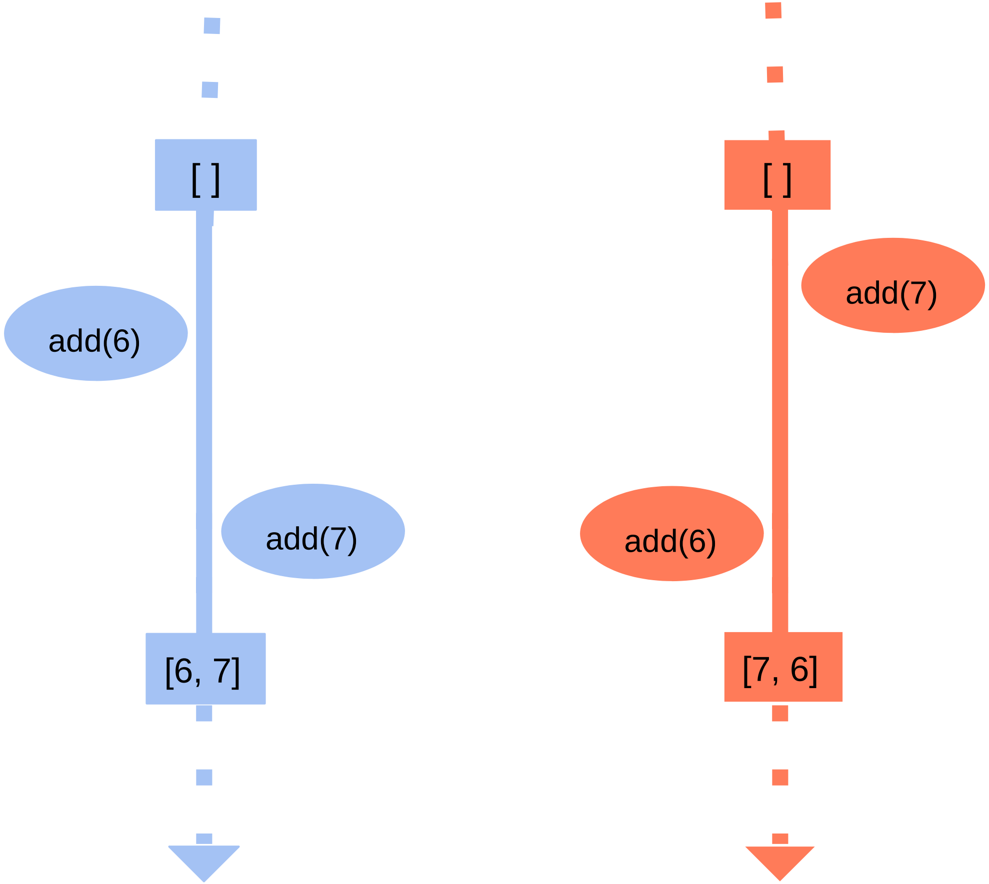

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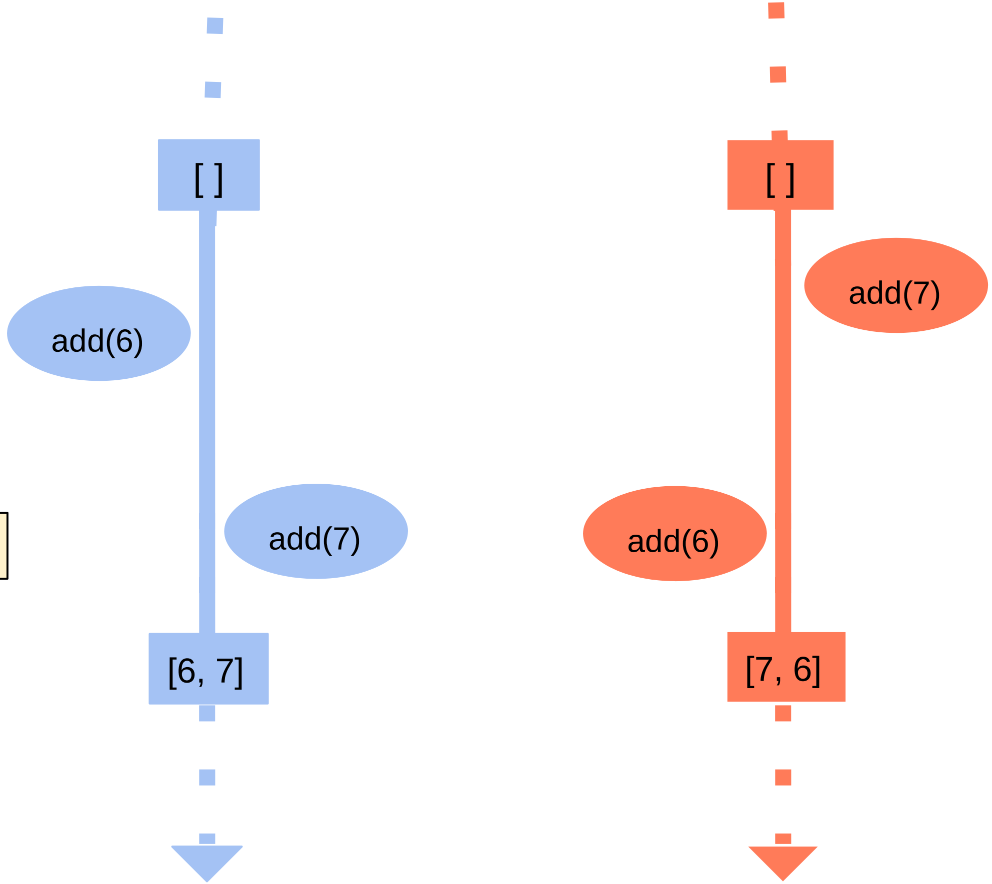
while i < h:          ||  while j < 100:
  i += 1              ||    j += 1
atomic:               ||  atomic:
  shared.add(6)       ||    shared.add(7)
             return sort(shared)
```

Abstraction **α**: list → multiset of elements



Commutativity Modulo Abstraction ("Abstract Commutativity")

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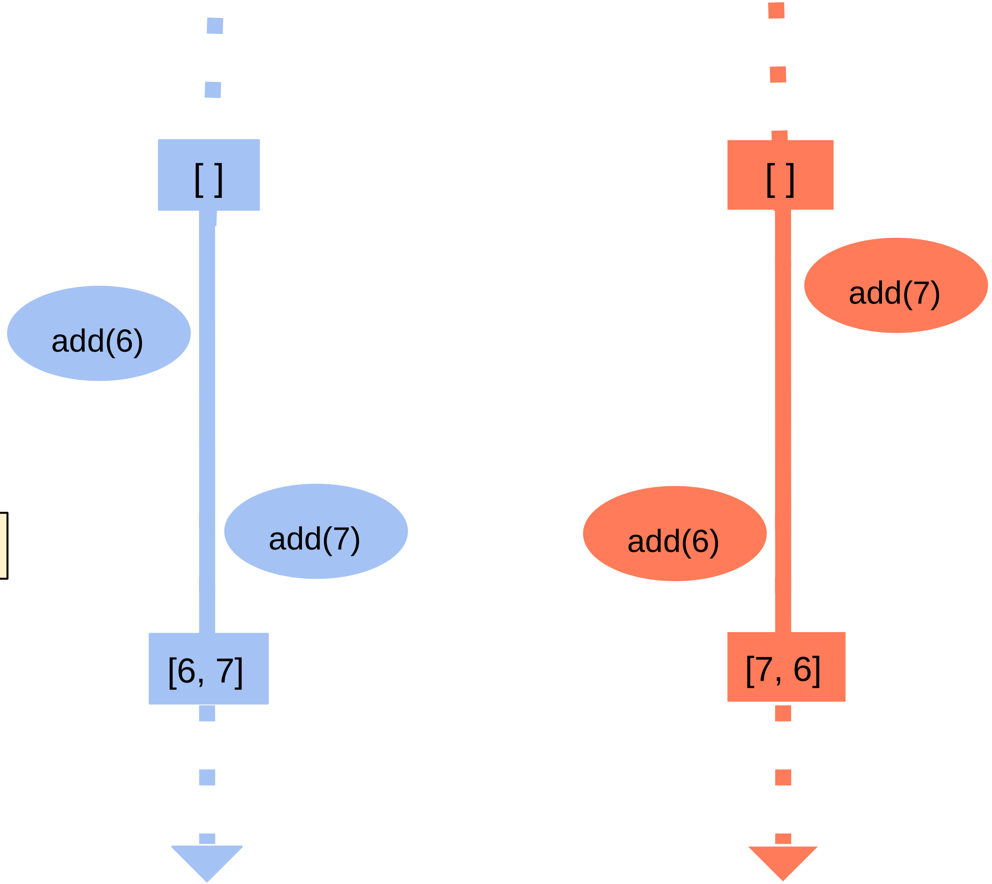

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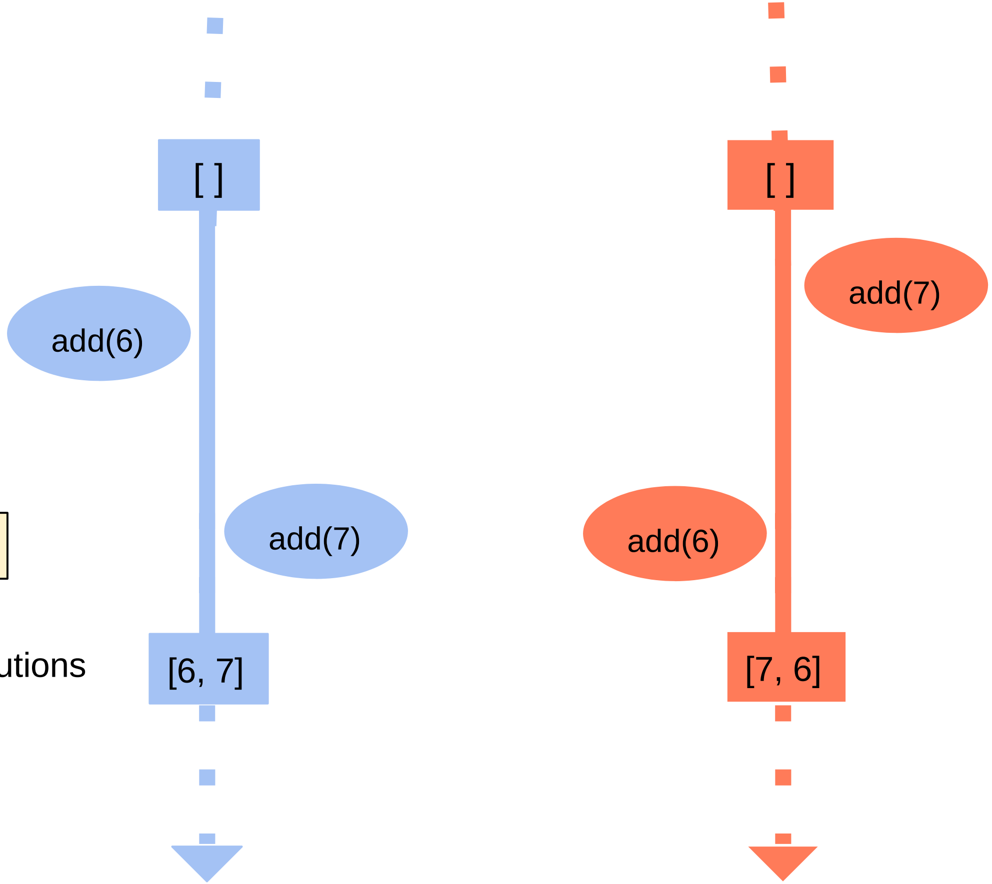

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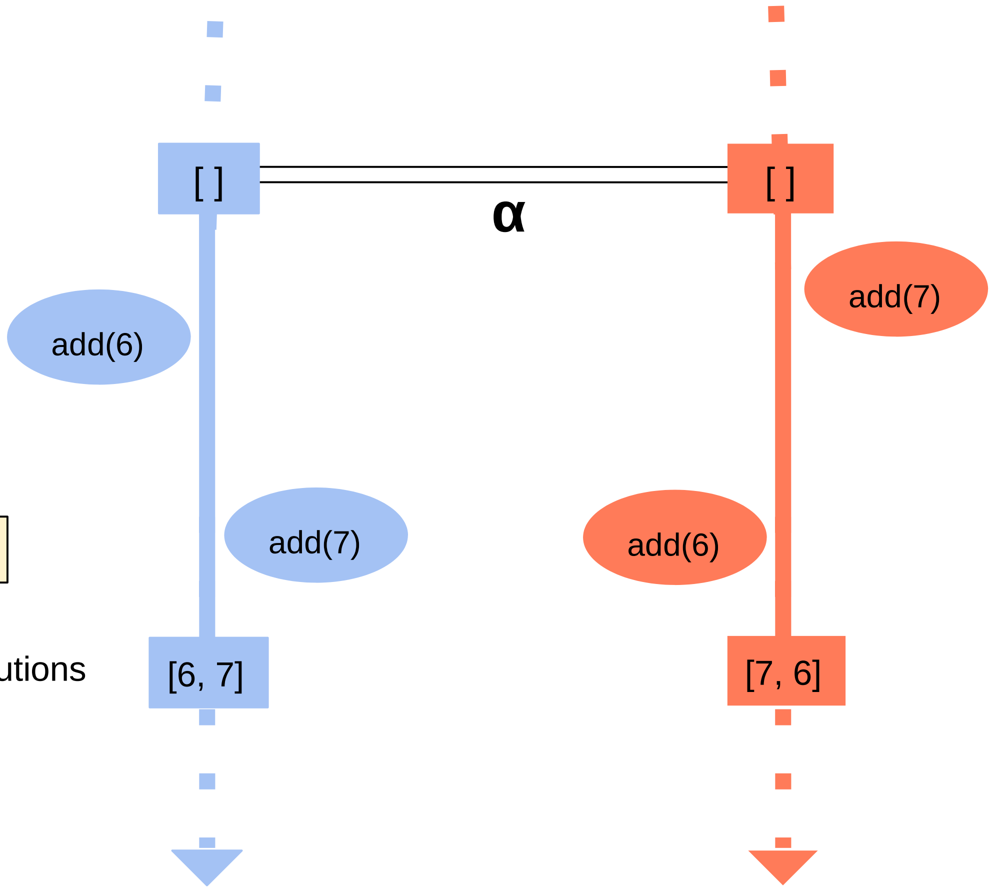

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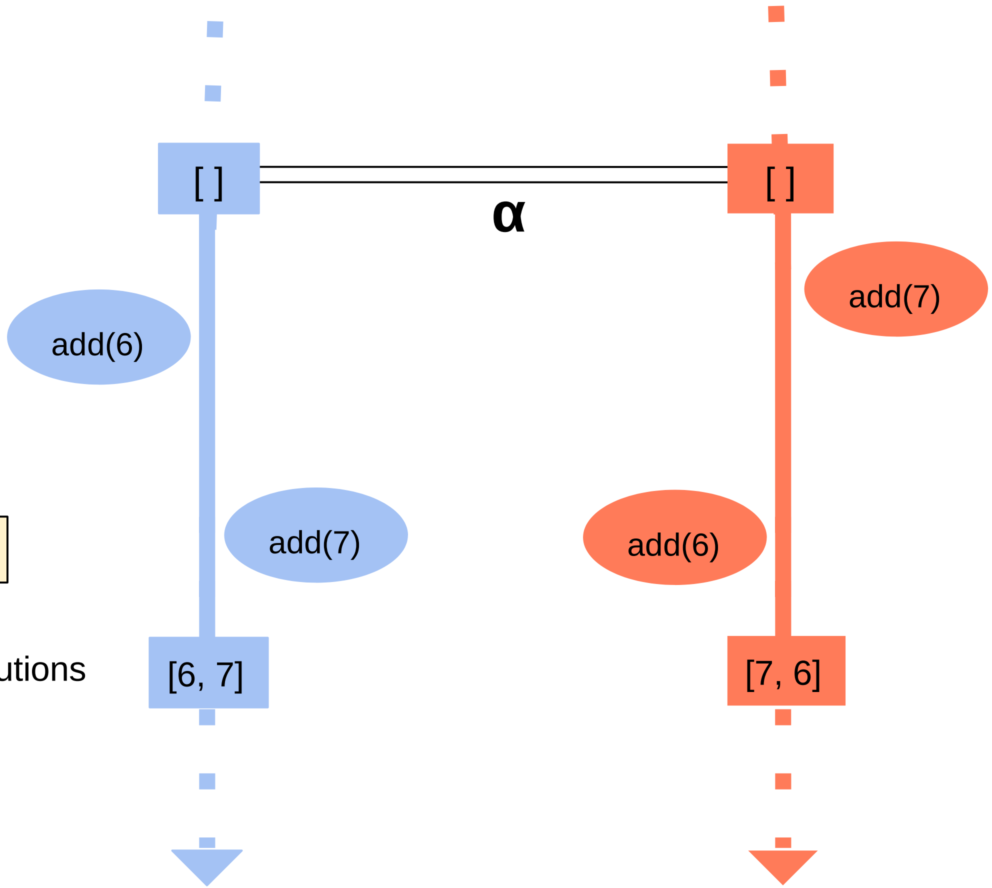

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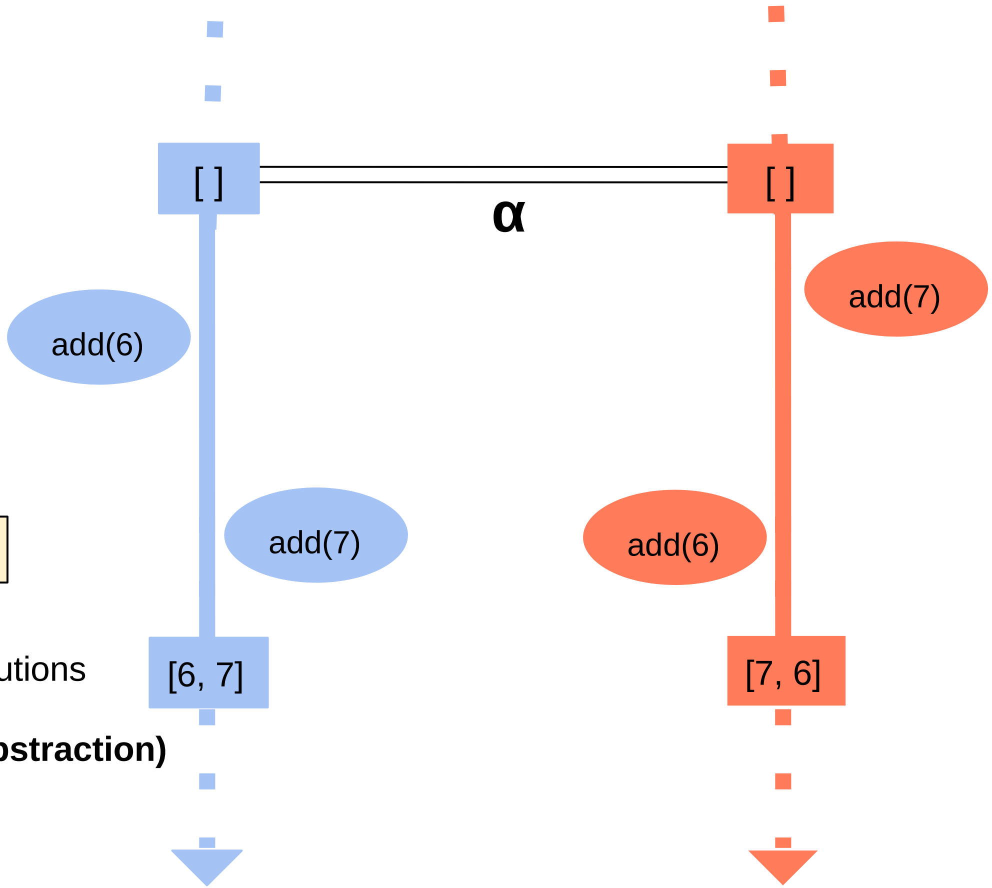

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(2) executions perform "same" modifications (**modulo abstraction**)

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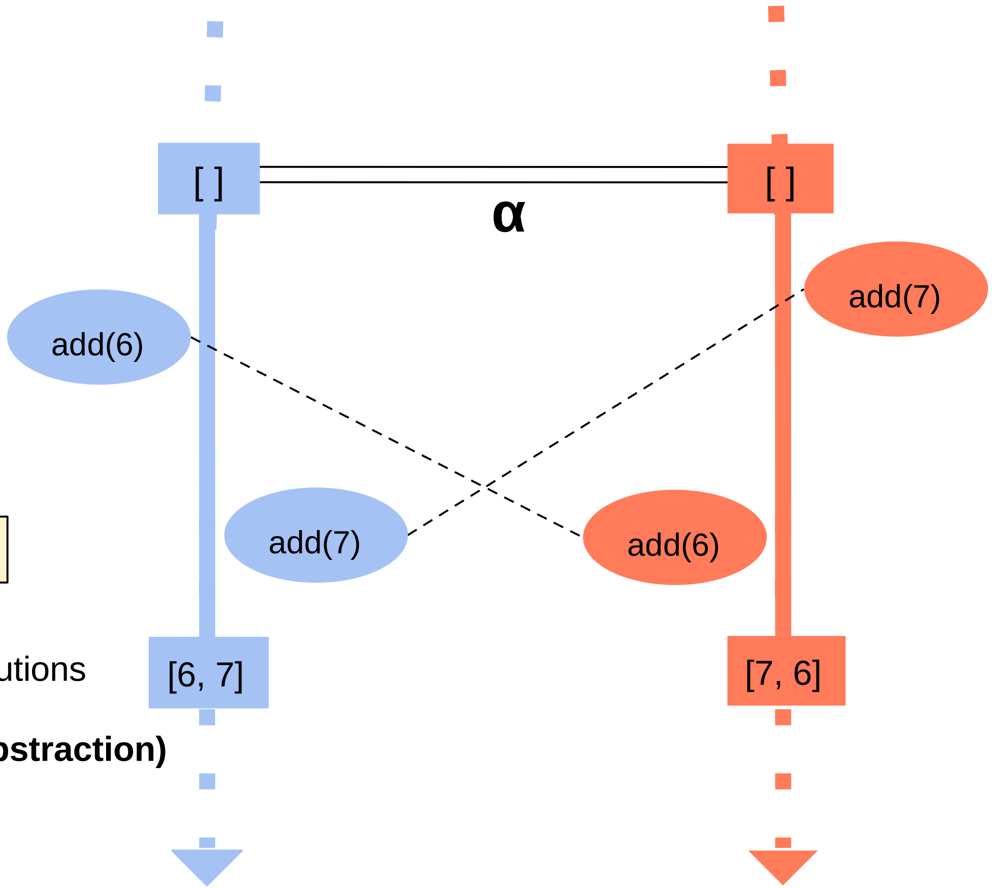

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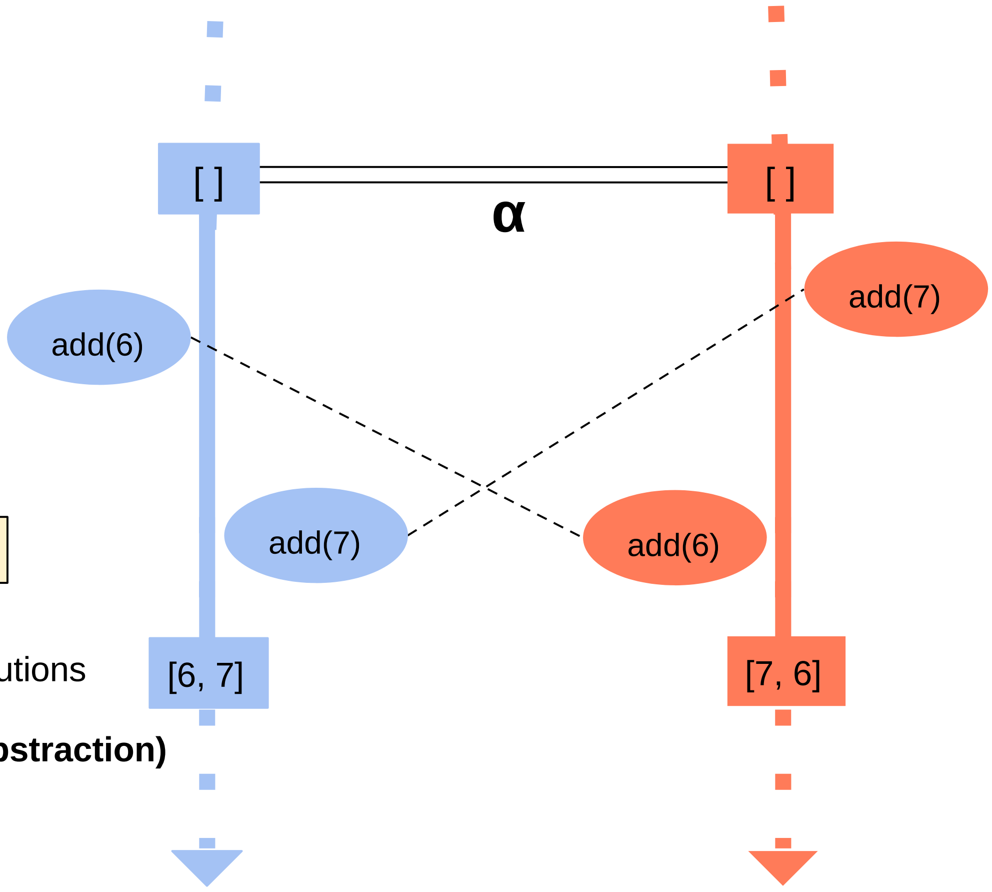

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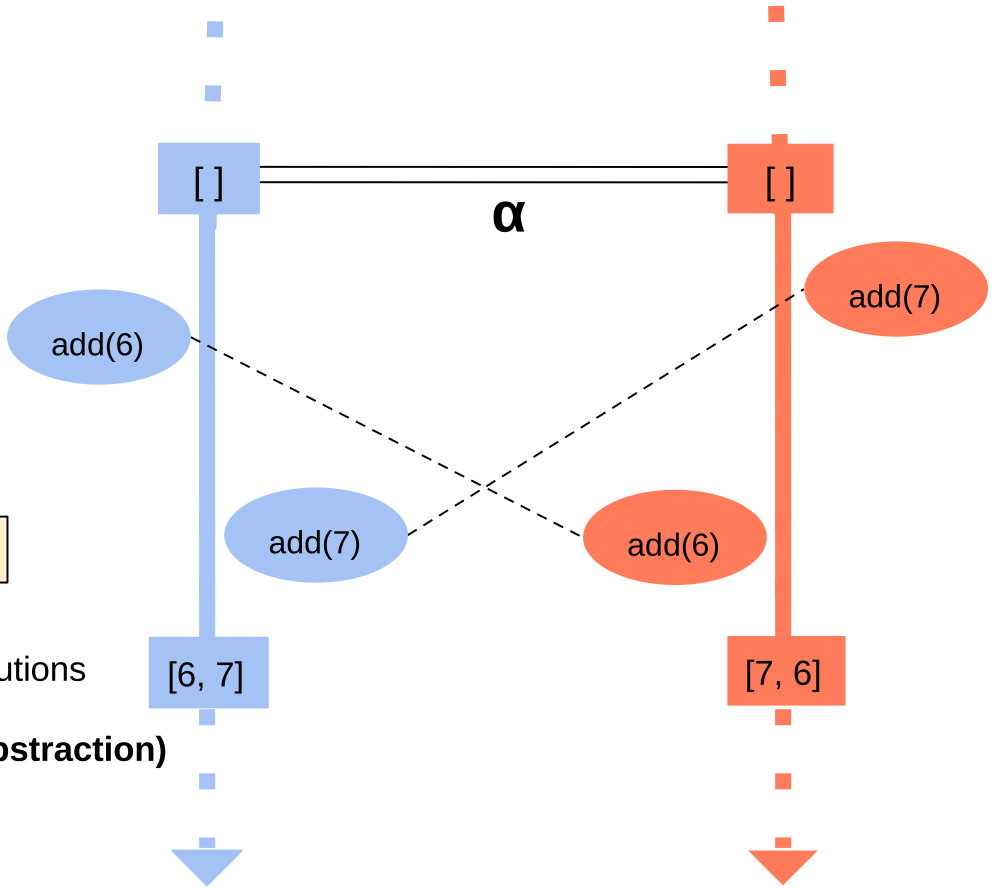

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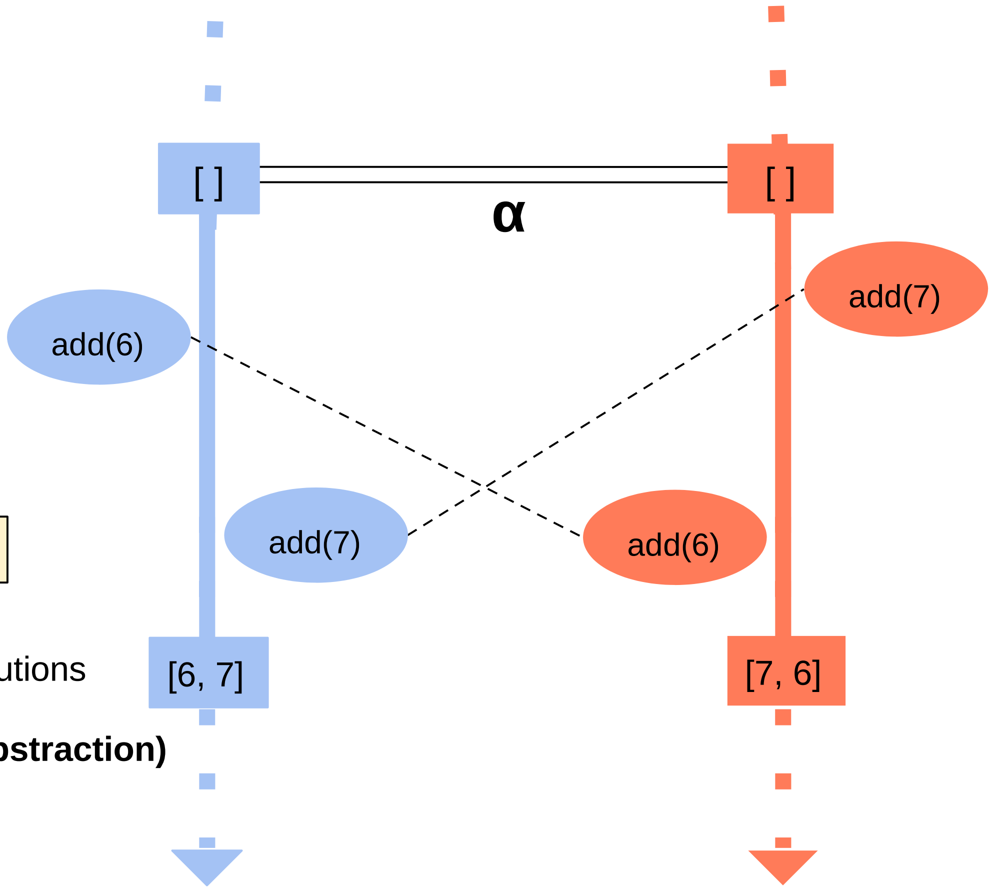

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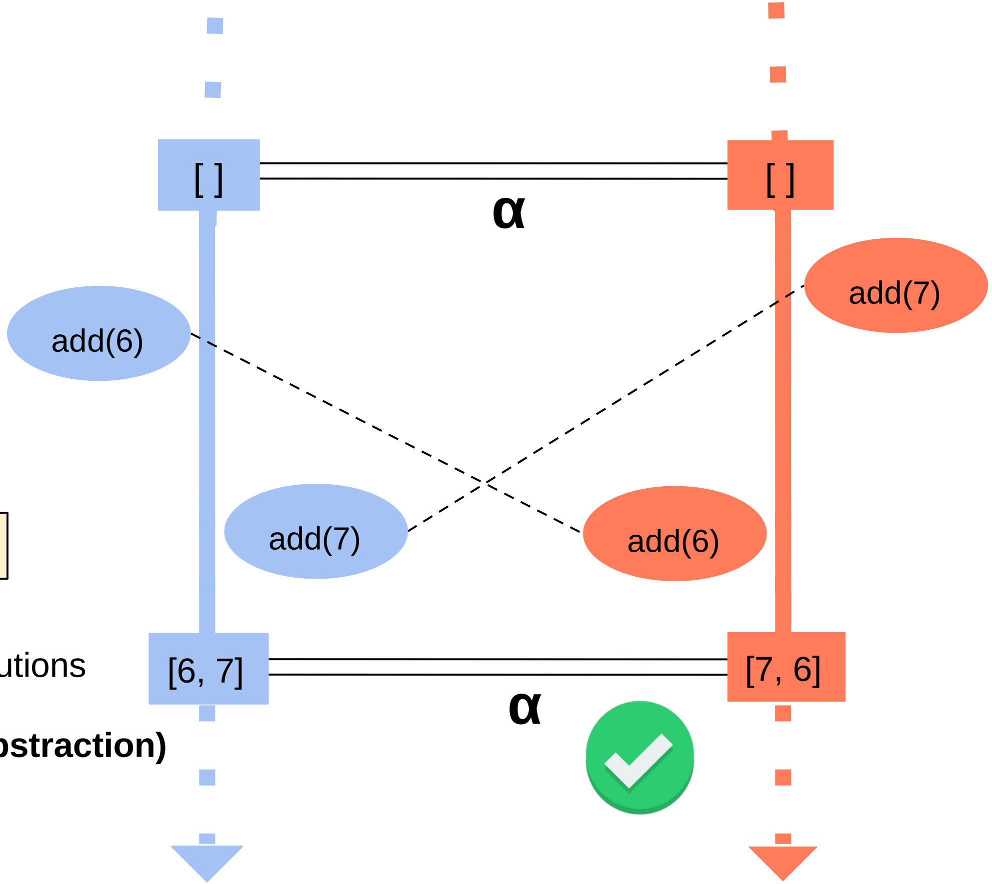

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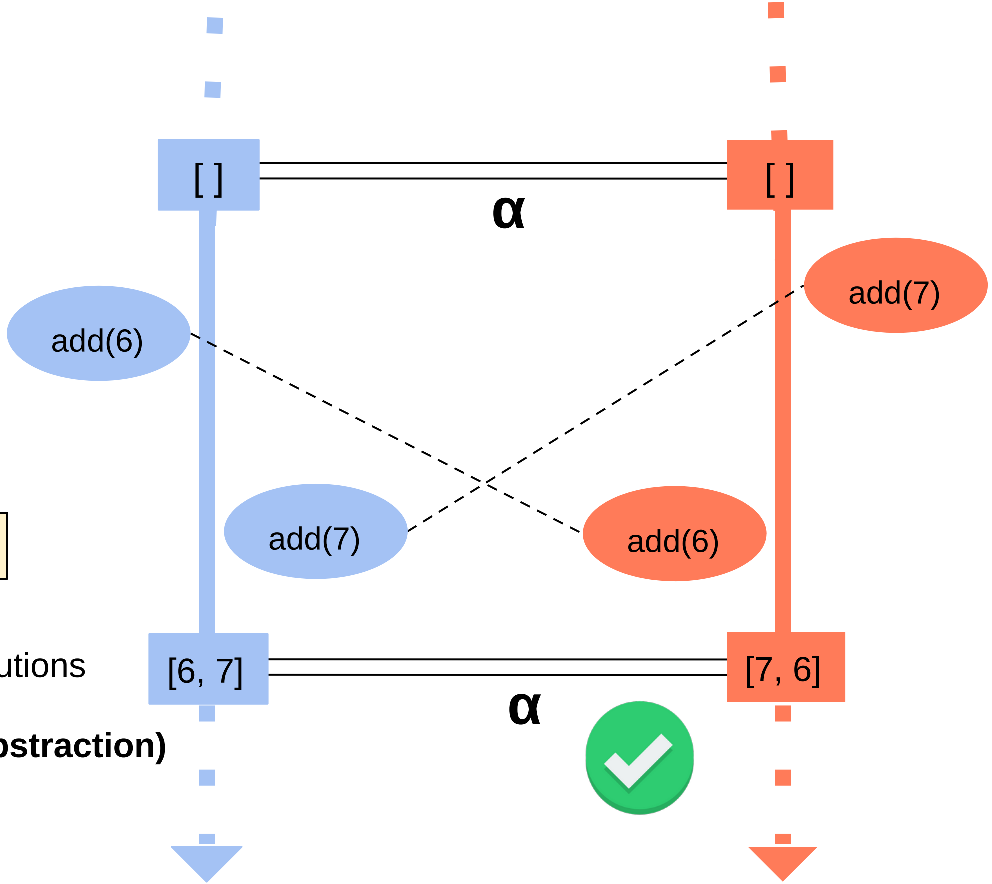

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Commutativity Modulo Abstraction (“Abstract Commutativity”)

Abstraction **α**: list $\rightarrow$ multiset of elements

Commutativity Modulo Abstraction (“Abstract Commutativity”)

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	Commutativity	**Commutativity modulo α**
f and *g* commute		
f and *g* are the “same”		

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return shared.keySet()
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CommCSL

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$$\Gamma \vdash \{P\}C\{Q\}$$

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Program

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$$\Gamma \vdash \{\underbrace{P}_{\text{Precondition}}\}\underbrace{C}_{\text{Program}}\{\underbrace{Q}_{\text{Postcondition}}\}$$

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Precondition

Postcondition

$$\Gamma \vdash \{P\}C\{Q\}$$

Invariant

Program

CommCSL

Precondition

Postcondition

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Invariant

Program

$$\frac{\Gamma_\perp = \Gamma \Rightarrow x \notin fv(\Gamma)}{\Gamma_\perp \vdash \{P[e/x]\}x{:=}e\{P\}}\ (\textsc{Assign}) \qquad \frac{x \notin fv(e) \quad \Gamma_\perp = \Gamma \Rightarrow x \notin fv(\Gamma)}{\Gamma_\perp \vdash \{\mathtt{emp}\}x{:=}\mathtt{alloc}(e)\{x \mapsto^1 e\}}\ (\textsc{New})$$

$$\frac{x \notin fv(e_1, e_2) \quad \Gamma_\perp = \Gamma \Rightarrow x \notin fv(\Gamma)}{\Gamma_\perp \vdash \{e_1 \mapsto^r e_2\}x{:=}[e]\{e_1 \mapsto^r e_2 * x = e_2\}}\ (\textsc{Read}) \qquad \frac{}{\Gamma_\perp \vdash \{e_1 \mapsto^1 \_\}[e_1]{:=}e_2\{e_1 \mapsto^1 e_2\}}\ (\textsc{Write})$$

$$\frac{\Gamma_\perp \vdash \{P \land b\}c_1\{Q\} \quad \Gamma_\perp \vdash \{P \land \neg b\}c_2\{Q\}}{\Gamma_\perp \vdash \{P \land \mathrm{Low}(b)\}\mathtt{if}\ (b)\ \mathtt{then}\ \{c_1\}\ \mathtt{else}\ \{c_2\}\{Q\}}\ (\textsc{If1})$$

$$\frac{\Gamma_\perp \vdash \{P \land b\}c_1\{Q\} \quad \Gamma_\perp \vdash \{P \land \neg b\}c_2\{Q\} \quad \mathit{unary}\ Q}{\Gamma_\perp \vdash \{P\}\mathtt{if}\ (b)\ \mathtt{then}\ \{c_1\}\ \mathtt{else}\ \{c_2\}\{Q\}}\ (\textsc{If2})$$

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$$\frac{\Gamma_\perp \vdash \{P\}c_1\{R\} \quad \Gamma_\perp \vdash \{R\}c_2\{Q\}}{\Gamma_\perp \vdash \{P\}c_1;c_2\{Q\}}\ (\textsc{Seq}) \qquad \frac{}{\Gamma_\perp \vdash \{P\}\mathtt{skip}\{P\}}\ (\textsc{Skip})$$

$$\frac{\begin{array}{c}\Gamma_\perp \vdash \{P_1\}c_1\{Q_1\} \quad \Gamma_\perp \vdash \{P_2\}c_2\{Q_2\} \quad fv(P_1, c_1, Q_1) \cap mod(c_2) = \emptyset \quad fv(P_2, c_2, Q_2) \cap mod(c_1) = \emptyset \\ \Gamma_\perp = \Gamma \Rightarrow fv(\Gamma) \cap mod(c_1, c_2) = \emptyset \quad P_1 \text{ is precise or } P_2 \text{ is precise}\end{array}}{}\ (\textsc{Par})$$

$$\frac{\ldots Q'\} \quad Q' \Rightarrow Q}{\Gamma_\perp \vdash \{P\}c\{Q\}}\ (\textsc{Cons}) \qquad \frac{\begin{array}{c}fv(R) \cap mod(c) = \emptyset \quad \Gamma_\perp \vdash \{P\}c\{Q\} \\ P \text{ is precise or } R \text{ is precise}\end{array}}{\Gamma_\perp \vdash \{P * R\}c\{Q * R\}}\ (\textsc{Frame})$$

$$\frac{x \notin fv(c) \quad \Gamma_\perp = \Gamma \Rightarrow x \notin fv(\Gamma) \quad \Gamma_\perp \vdash \{P\}c\{Q\}}{\Gamma_\perp \vdash \{\exists x.\, P\}c\{\exists x.\, Q\}}\ (\textsc{Exists})$$

$$\frac{\begin{array}{c}\Gamma = \langle \alpha, f_{a_s}, f_{a_u}, I(x)\rangle \quad \Gamma \text{ is valid} \quad I(x) \text{ is unary and precise} \\ \Gamma \vdash \{P * \mathit{sguard}(1, \emptyset^\#) * \mathit{uguard}([])\}c\{Q * \mathit{sguard}(1, x_s) * \mathit{PRE}_s(x_s) * \mathit{uguard}(x_u) * \mathit{PRE}_u(x_u)\}\end{array}}{\perp \vdash \{I(x) * \mathrm{Low}(\alpha(x)) * P\}c\{\exists x'.\, I(x') * \mathrm{Low}(\alpha(x')) * Q\}}\ (\textsc{Share})$$

$$\frac{\begin{array}{c}\Gamma = \langle \alpha, f_{a_s}, f_{a_u}, I(x)\rangle \quad I(x_v) \text{ is unary and precise} \\ x_v \notin fv(P, Q) \quad x_s, x_a, x_v \notin mod(c) \quad \mathit{noguard}(P) \quad \mathit{noguard}(Q) \\ \perp \vdash \{P * I(x_v)\}c\{Q * I(f_{a_s}(x_v, x_a))\}\end{array}}{\Gamma \vdash \{P * \mathit{sguard}(r, x_s)\}\mathtt{atomic}\ c\{Q * \mathit{sguard}(r, x_s \cup^\# \{x_a\}^\#)\}}\ (\textsc{AtomicShr})$$

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CommCSL

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- Relational concurrent separation logic
- Support for (abstract) commutativity-based information flow reasoning
- Thread-modular reasoning, mutable heaps

- Other features:
  - Low events, standard output…
  - More complete support for non-symmetric concurrency

- Formalized and proved sound in Isabelle/HOL
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Non-interference theorem

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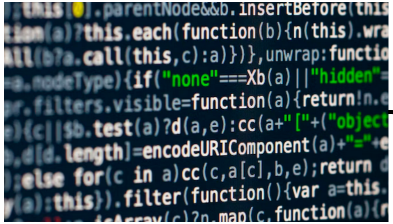
Implementation

HyperViper

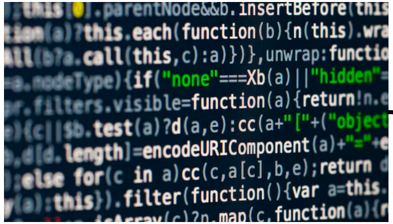
Implementation



Source code

HyperViper

Implementation



Source code

Specification
(e.g., low variables and data)

HyperViper

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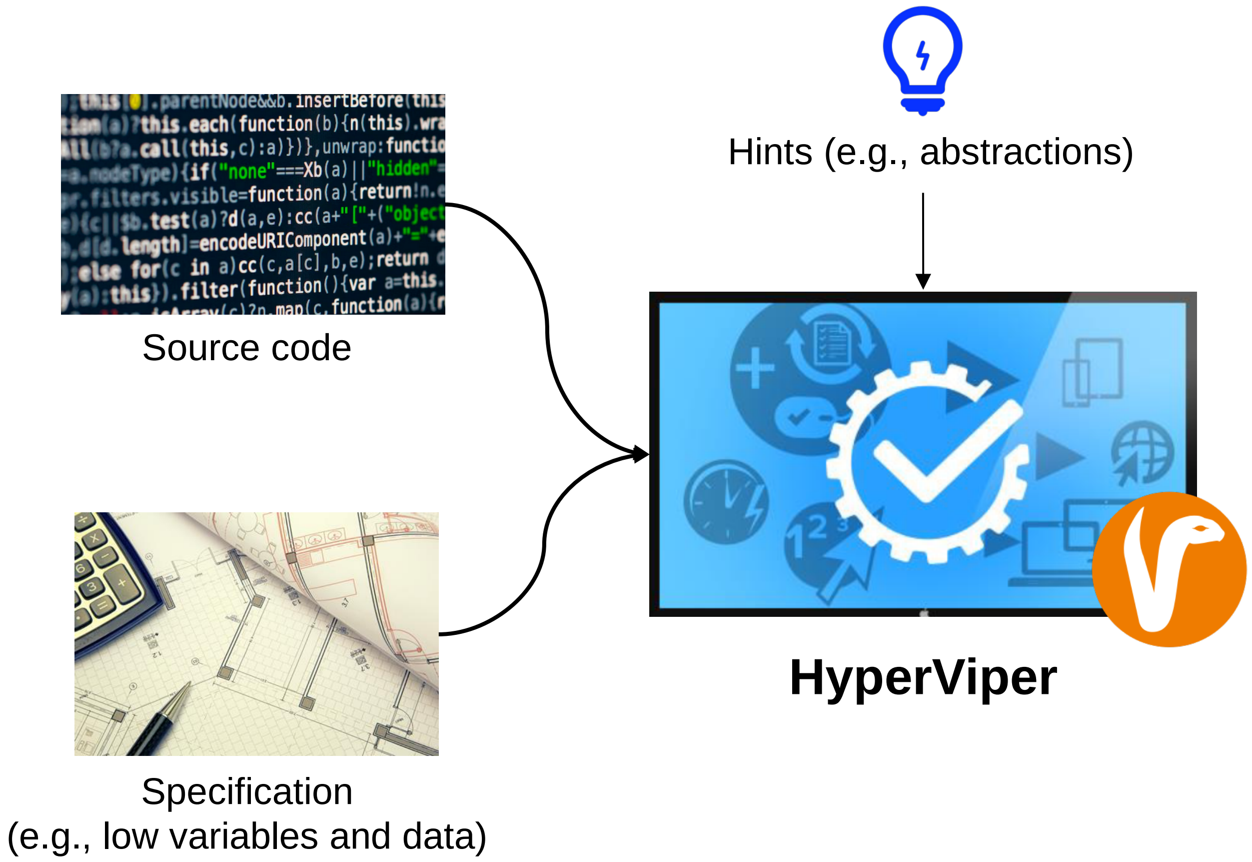
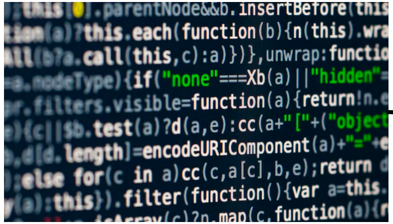


Implementation

Hints (e.g., abstractions)



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HyperViper

No information leak through values
(in all executions)

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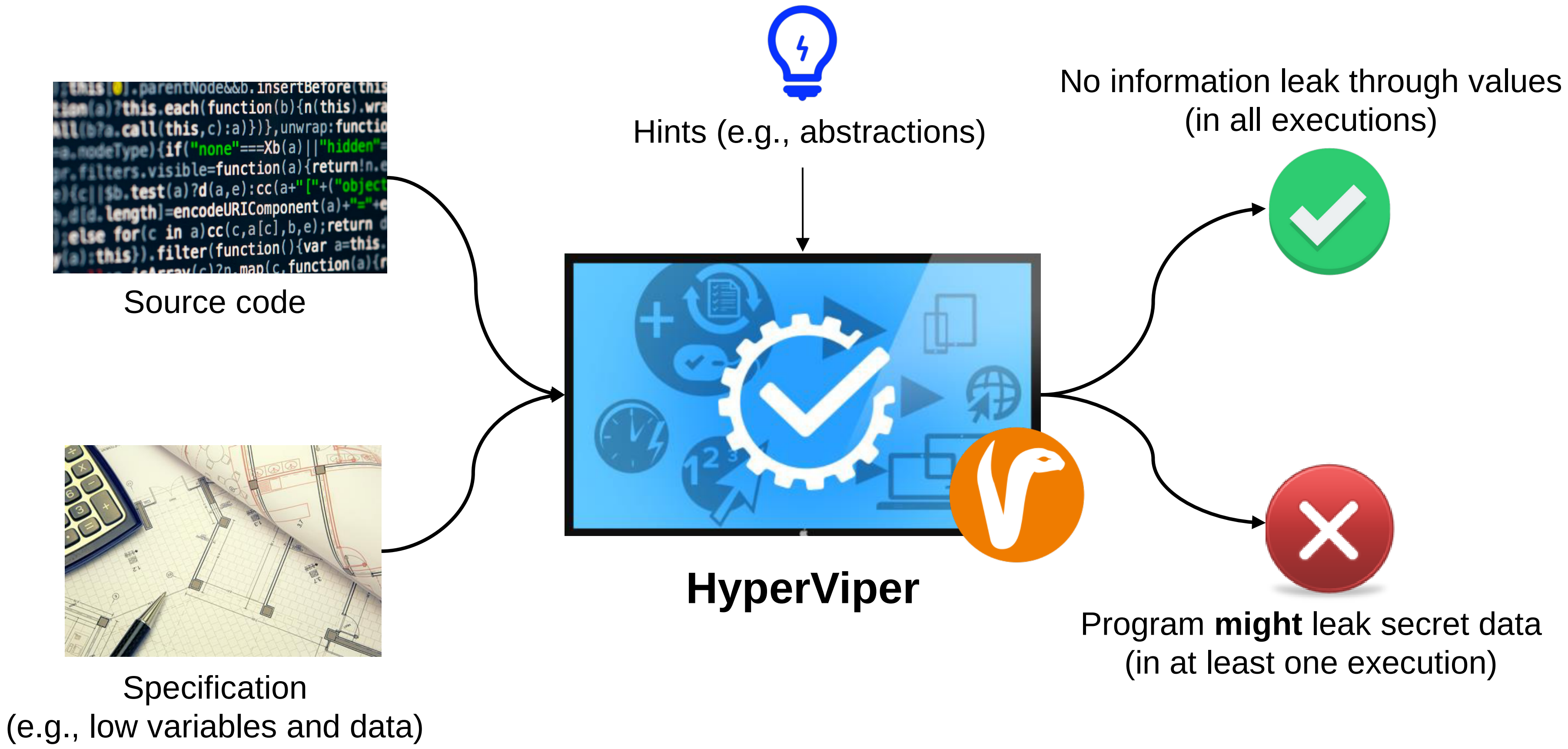

HyperViper

- Automated, SMT-based verifier
  - Based on Viper verification infrastructure and Z3
  - Relational reasoning using Modular Product Programs
- User provides abstractions, pre- and postconditions, invariants…
- Supports dynamic thread creation, multiple shared resources, …
- https://github.com/viperproject/hyperviper

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```
lockType IntLock {
 type Int
 invariant(l, v) = [l.lockInt |-> ?cp && [cp.val |-> v]]
 alpha(v): Int = 0  // we abstract to a constant, so everything commutes
 actions = [(SetValue, Int, duplicable)]
 action SetValue(v, arg)
   requires true
 { arg }
 noLabels = 2
}

...

method worker(l: Lock, lbl: Int)
 requires lowEvent && sguard[IntLock,SetValue](l, Set(lbl))
 requires sguardArgs[IntLock,SetValue](l, Set(lbl)) == Multiset[Int]()
 ensures sguard[IntLock,SetValue](l, Set(lbl))
 ensures allPre[IntLock, SetValue](sguardArgs[IntLock,SetValue](l, Set(lbl)))
{

   var v: Int
   v := lbl
   with[IntLock] l performing SetValue(v) at lbl {
       l.lockInt.val := v
   }

}

method print(i: Int)
 requires lowEvent && low(i)
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Evaluation

Example	Data structure	Abstraction	LOC	Ann.	T
Count-Vaccinated	Counter, increment	None	44	46	10.15
Figure 2	Integer, add	None	129	95	10.90
Count-Sick-Days	Integer, add	None	52	45	13.67
Figure 1	Integer, arbitrary	Constant	29	20	1.52
Mean-Salary	List, append	Mean	80	84	14.10
Email-Metadata	List, append	Multiset	82	75	16.70
Patient-Statistic	List, append	Length	73	70	4.92
Debt-Sum	List, append	Sum	76	81	14.45
Sick-Employee-Names	Treeset, add	None	105	113	28.43
Website-Visitor-IPs	Listset, add	None	74	69	6.20
Figure 3	HashMap, put	Key set	129	96	10.37
Sales-By-Region	HashMap, disjoint put	None	129	104	12.37
Salary-Histogram	HashMap, increment value	None	135	109	13.78
Count-Purchases	HashMap, add value	None	137	109	11.73
Most-Valuable-Purchase	HashMap, conditional put	None	140	118	17.87
1-Producer-1-Consumer	Queue	Consumed sequence	82	88	3.23
Pipeline	Two queues	Consumed sequences	122	100	3.66
2-Producers-2-Consumers	Queue	Produced multiset	130	134	8.45

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Secret data influences which thread performs which modification

- Modular reasoning about value sensitivity for concurrent programs
  - Independently of timing
  - Sound on real hardware
- Key idea is to exploit commutativity modulo abstraction
- Proved sound in Isabelle/HOL, automated in prototype verifier
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Thank you for your attention!

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